

Millikan Oil Drop Experiment

Physics 401, Fall 2019

Eugene V. Colla



Agenda

- 1. Measuring of the charge of electron.**
- 2. Robert Millikan and his oil drop experiment**
- 3. Theory of the experiment**
- 4. Laboratory setup**
- 5. Data analysis**



Measuring of the charge of the electron

1. **Oil drop experiment. Robert A. Millikan.. (1909).**
 $e=1.5924(17) \times 10^{-19} \text{ C}$
2. **Shot noise experiment. First proposed by Walter H. Schottky**
3. **In terms of the Avogadro constant and Faraday constant $e = \frac{F}{N_A}$; F- Faraday constant, N_A - Avogadro constant. Best uncertainty ~ 1.6 ppm.**
4. **From Josephson ($K_J = \frac{2e}{h}$) and von Klitzing ($R_K = \frac{h}{e^2}$) constants**
5. **Recommended by NIST value $1.602\ 176\ 565(35) \ 10^{-19} \text{ C}$**



Robert Millikan. Oil Drop Experiment



**ROBERT ANDREWS
MILLIKAN
1868-1953**



**The Nobel Prize in Physics 1923.
Robert A. Millikan "for his work on the
elementary charge of electricity and on
the photoelectric effect".**

Vol. II.] *ELECTRICAL CHARGE AND AVOGADRO CONSTANT.* 109
No. 2.]

ON THE ELEMENTARY ELECTRICAL CHARGE AND THE
AVOGADRO CONSTANT.

BY R. A. MILLIKAN.
I. INTRODUCTORY.

**22nd of March,
1868, Morrison, Ill**

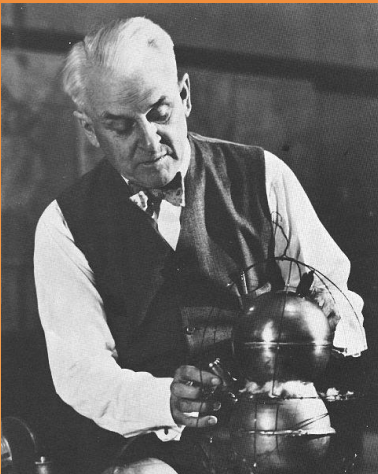


University of Chicago

9/23/2019

**On January 26, 1982, he was honored
by the United States Postal Service
with a 37¢ Great Americans series
(1980–2000) postage stamp.**

Robert Millikan. Oil drop experiment



**ROBERT ANDREWS
MILLIKAN
1868-1953**

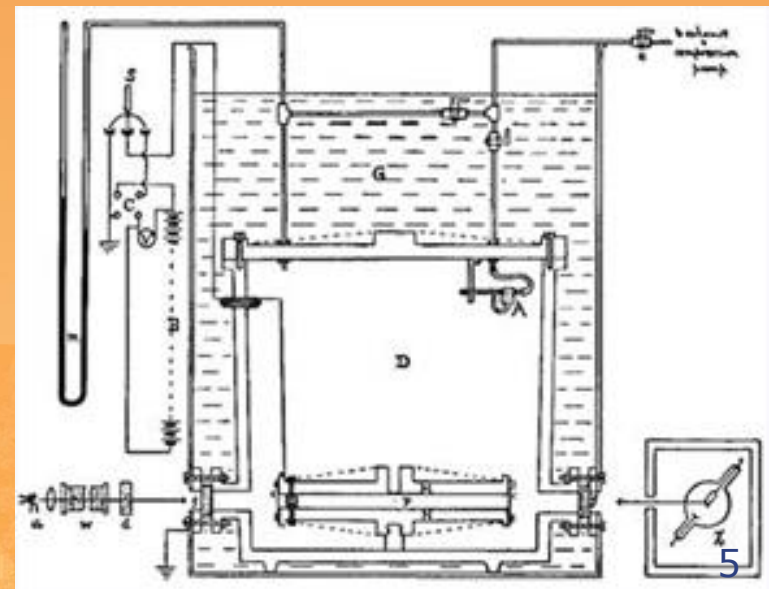


Diagram and picture of apparatus

Oil Drop Experiment.

Motivation:

Measurement of the magnitude of the electron charge!

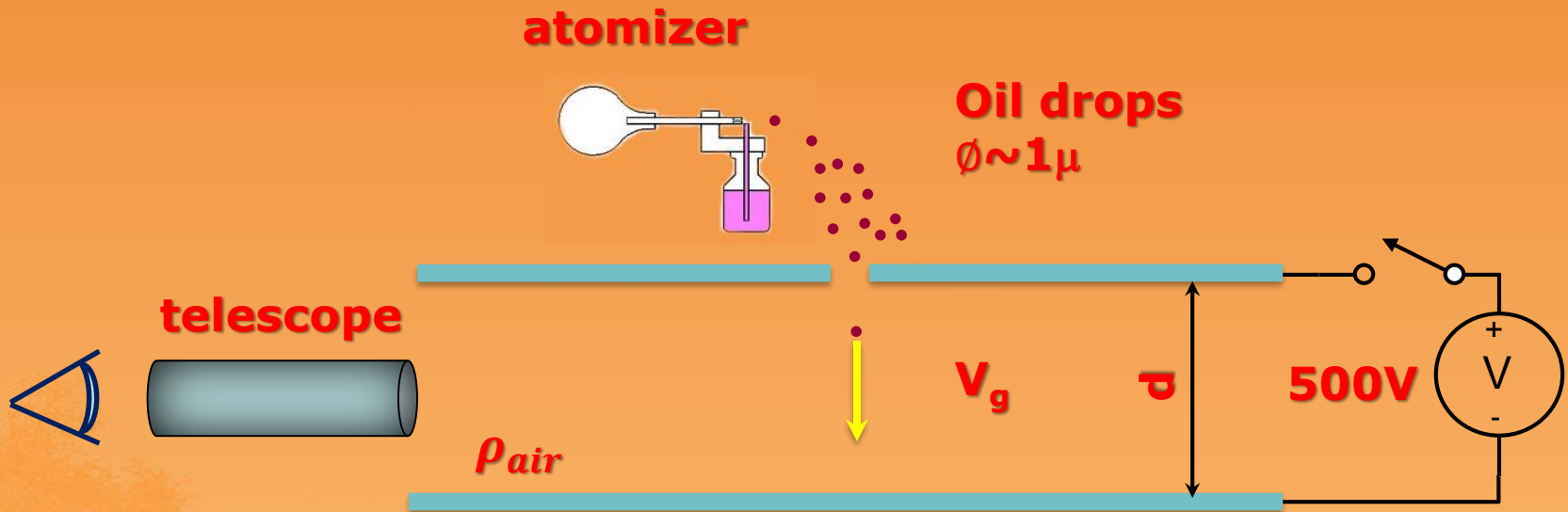
Demonstrate that the electron charge is quantized!



Measure the charge of
an electron to $\pm 3\%$

Picture of the PASCO setup

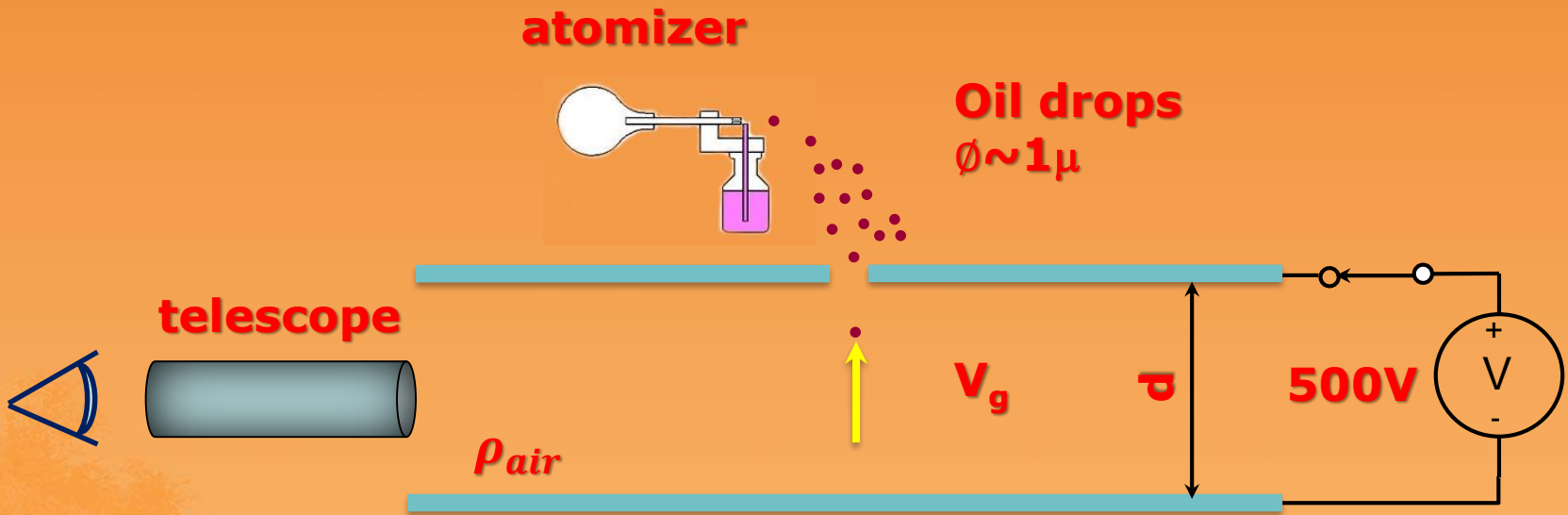
Oil Drop Experiment.



Forces on the oil drop:

- 1) Gravity + buoyant force (air displaced by oil drop)
- 2) Drag force of the oil drop in the air

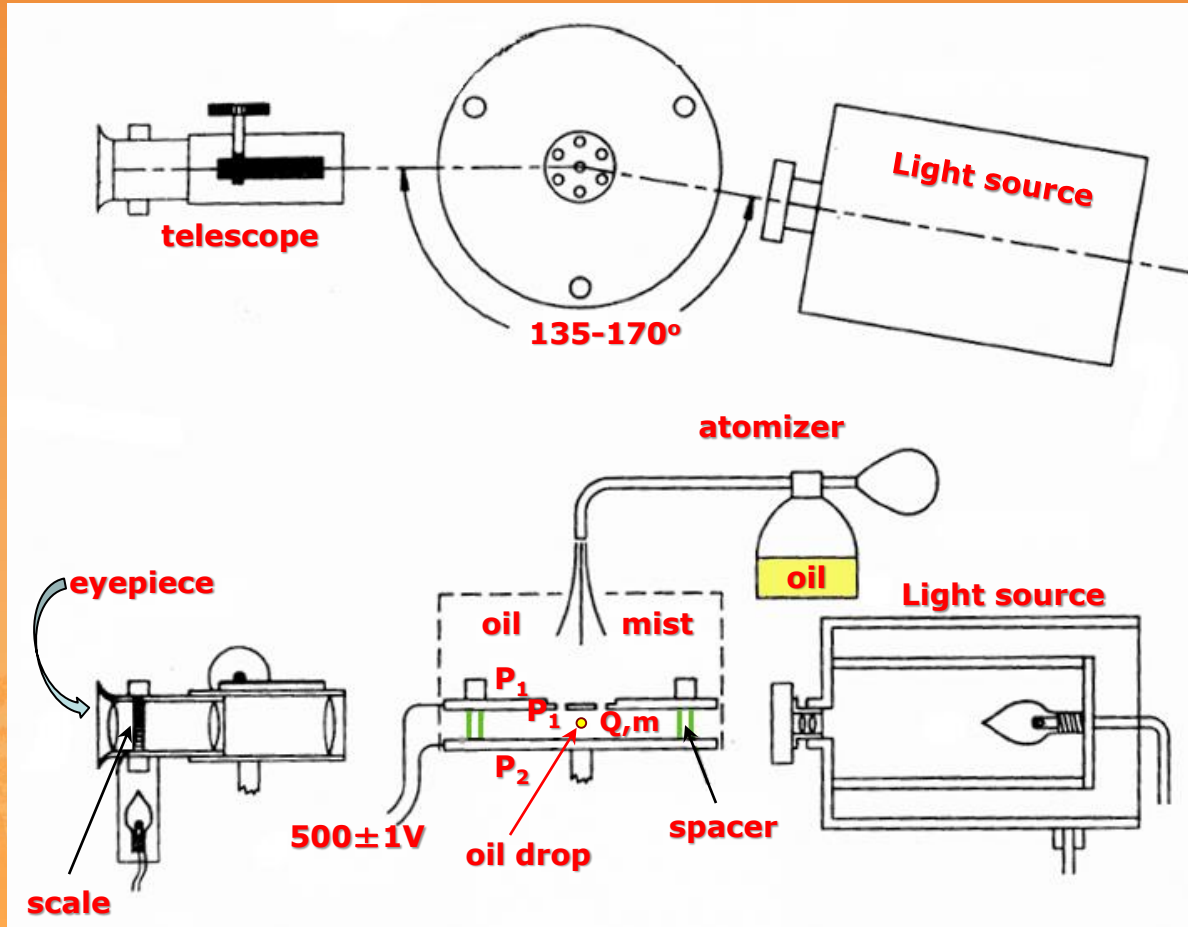
Oil Drop Experiment.



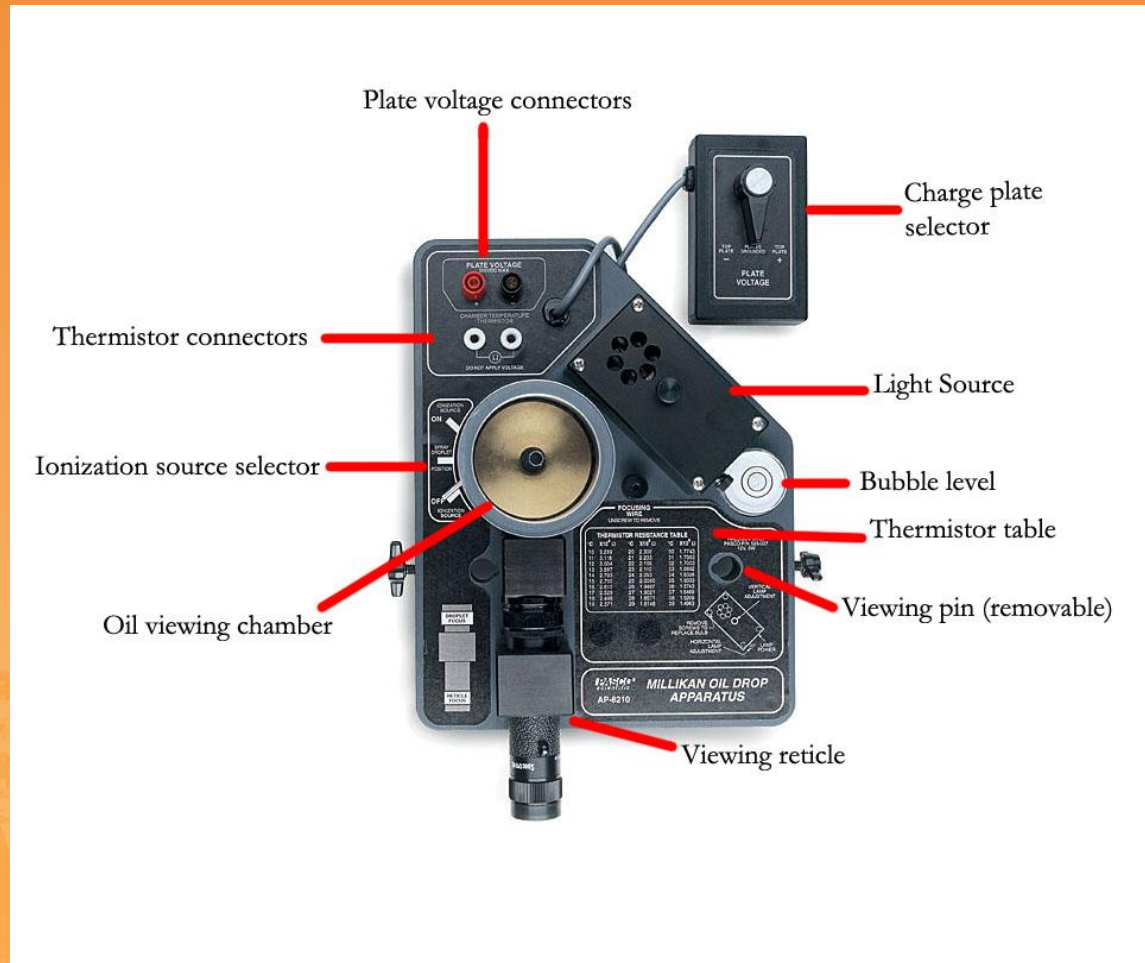
Forces on the oil drop:

- 1) Gravity + buoyant force (air displaced by oil drop)
- 2) Drag force of the oil drop in the air
- 3) Electric force on oil drops which carry charge Q

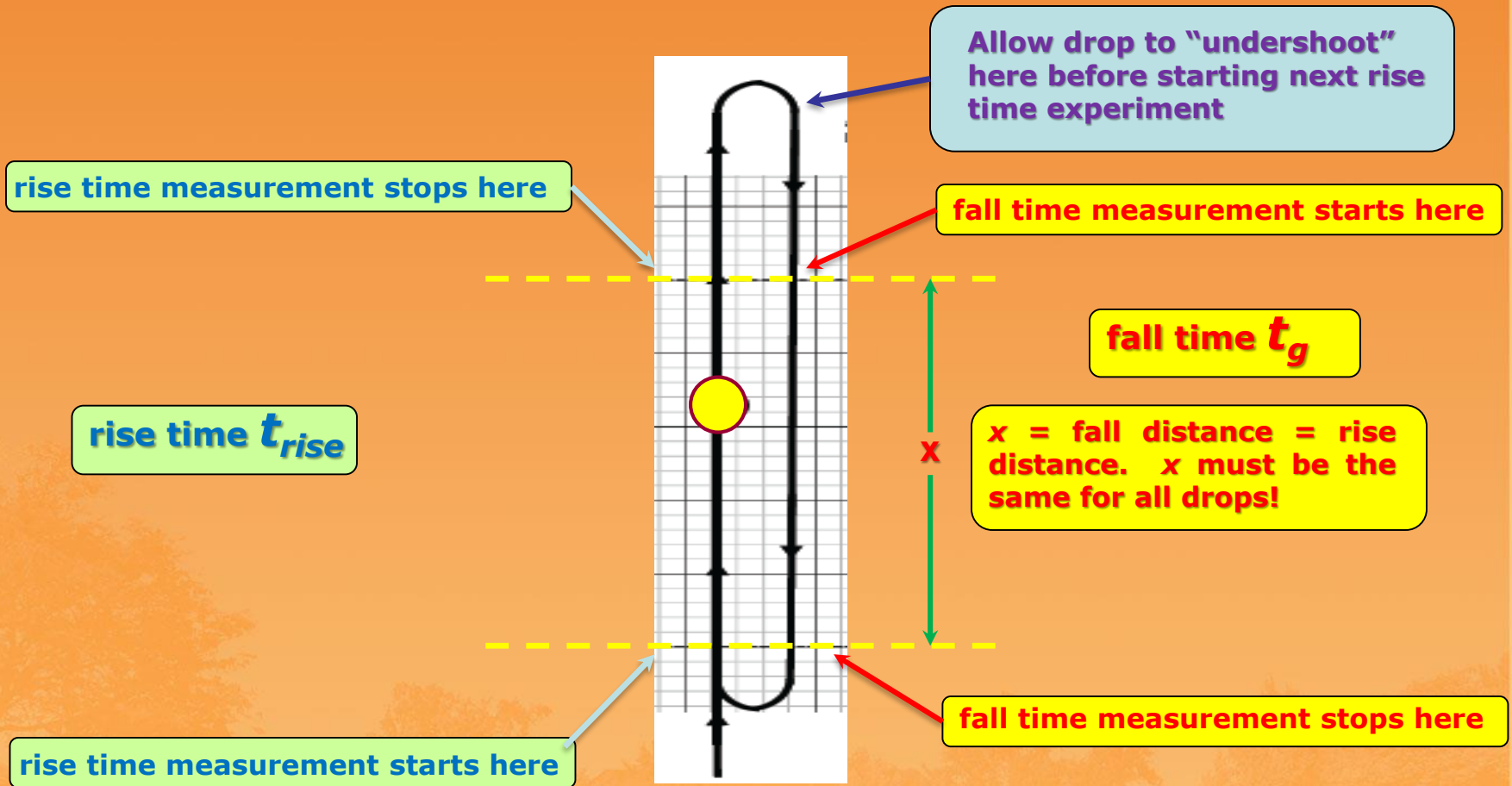
Apparatus. Schematic Layout



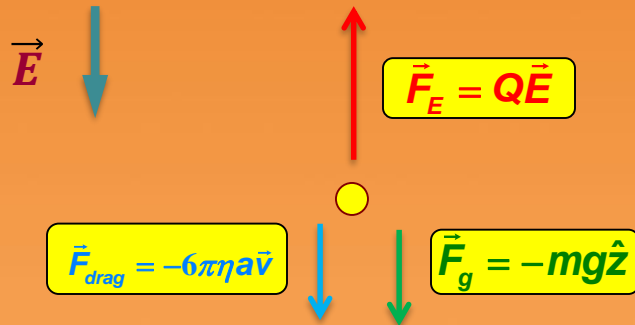
Apparatus: Actual Setup



What is Measured



Balance of Forces: Newton's Law



a : radius of drop
 ρ : density $\rho = \rho_{oil} - \rho_{air}$
 v : velocity of oil drop
 Q : charge of oil drop
 E : electric field $E=V/d$
 V : Voltage across plates
 η : viscosity of air
 g : gravitational const.

Particle reached terminal velocity $\frac{d\vec{v}}{dt} = 0$

$$\vec{F}_g + \vec{F}_{drag} + \vec{F}_E = 0$$

$$\vec{F}_g = -mg\hat{z} \quad (1)$$

$$\vec{F}_{drag} = -6\pi\eta a\vec{v} \quad (2)$$

$$\vec{F}_E = Q\vec{E} \quad (3)$$

$$\vec{F} = m \frac{d\vec{v}}{dt} = \vec{F}_g + \vec{F}_{drag} + \vec{F}_E$$

Forces on the oil drop:

- (1) Gravity + buoyant force (air displaced by oil drop)
- (2) Drag force of the oil drop in the air
- (3) Electric force on oil drops which carry charge Q

1μ size particle reaches the terminal velocity in $\sim 10^{-5}s$

Modification to Stokes Law



George Gabriel Stokes
(1819-1903)

$$\vec{F}_{drag} = -6\pi\eta a\vec{v}$$

For small particle radius ($a < 15\mu$) Stokes law need to be corrected. This correction was derived by E. Cunningham.



Ebenezer Cunningham
(1881-1977)

$$\vec{F}_{drag} = -6\pi\eta \frac{a}{f_c} \vec{v}$$

$$f_c = 1 + A\frac{\lambda}{a} + B\frac{\lambda}{a}e^{-C\frac{a}{\lambda}}, \quad A = 1.246, \quad B = 0.42, \quad C = 0.78$$

$$f_c \approx 1 + A\frac{\lambda}{a} = 1 + \frac{r_c}{a} \approx 1.1, \quad \text{for } a \approx 10^{-6}\text{m}, \quad r_c [m] = \frac{6.18 \times 10^{-5}}{\rho [\text{mmHg}]}$$

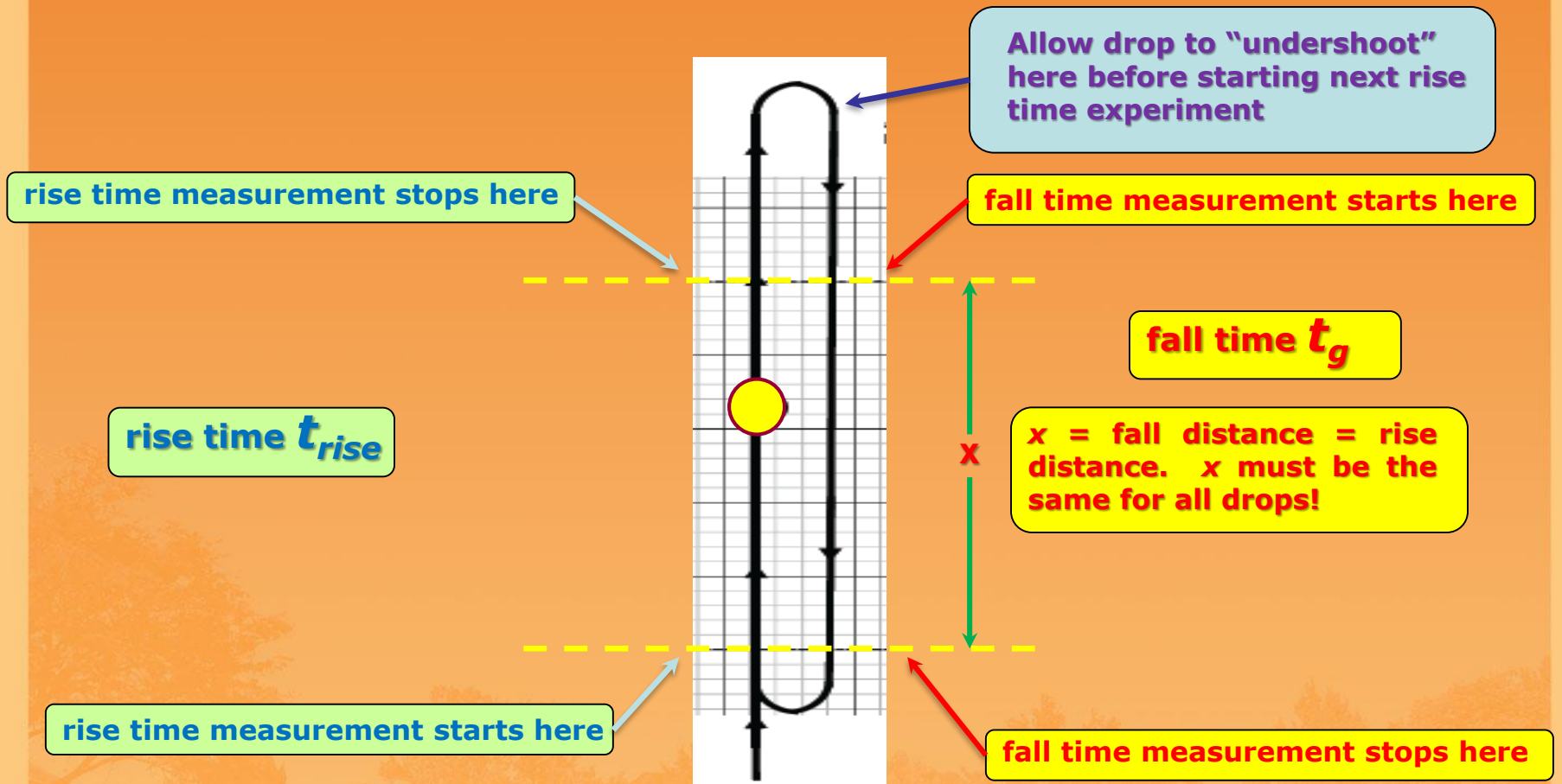
Here **a** – particle radius; **λ** – mean free path of the gas molecules

negligible term

$$\lambda [m] = 6.53 \times 10^{-8} \frac{760}{\rho [\text{mmHg}]}$$



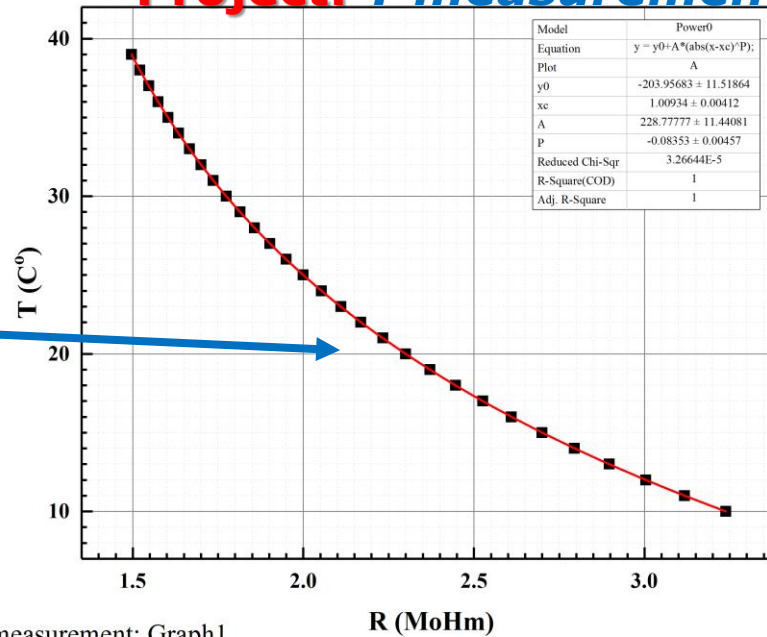
We Measure: t_g and t_{rise}



Measuring the Temperature



Project: *T measurement.opj*



T measurement: Graph1

Project Explorer (1)			A(X)	B(Y)
T measurement				
Folder1				
Name	Long Name	Short N	Long Name	Units
Tcalculation		Tcalcul	R	T
Graph1		Graph1	MoHm	Co
Book1		Book1		
			Comments	
			F(x)=	
			1.55	36.87987

Solving Newton's Law: $Q(t_g, t_{\text{rise}})$

f_c can be found from Newton law equation in the case of $V=0$ (falling drop)

$$\vec{F}_g + \vec{F}_{\text{drag}} = \frac{4}{3} a^3 \rho g - 6\pi\eta \frac{a}{f_c} \vec{v} = 0$$



(see write-up)

$$\frac{1}{f_c^3} \approx 1 - \left(\frac{t_g}{\tau_g} \right)^2; \quad \tau_g = \frac{2\eta x}{\rho g r_c^2}; \quad r_c[m] = \frac{6.18 \times 10^{-5}}{\rho[\text{mmHg}]}$$



Solving Newton's law: $Q(t_g, t_{rise})$

$$Q = n \times e = \frac{1}{f_c^{3/2}} \frac{9\pi d}{V} \sqrt{\frac{2\eta^3 x^3}{g\rho}} \sqrt{\frac{1}{t_g} \left[\frac{1}{t_g} + \frac{1}{t_{rise}} \right]}$$

Q : charge of oil drop
 n : number of unpaired electrons in drop
 e : elementary charge
 d : plate separation
 V : Voltage across plates

ρ : density $\rho = \rho_{oil} - \rho_{air}$
 η : viscosity of air
 g : gravitational constant
 x : drift distance for oil drop
 t_g : fall time
 t_{rise} : rise time



Route of Charge Calculation $Q(t_g, t_{rise})$.

$$\frac{1}{f_c^3} \approx 1 - \left(\frac{t_g}{\tau_g}\right)^2; \quad \tau_g = \frac{2\eta x}{\rho g r_c^2}; \quad r_c[m] = \frac{6.18 \times 10^{-5}}{\rho[mmHg]}$$

$$Q = F \cdot S \cdot T = \left(\frac{1}{f_c^{3/2}}\right) \frac{9\pi d}{V} \sqrt{\frac{2\eta^3 x^3}{g\rho}} \frac{1}{\sqrt{t_g}} \left(\frac{1}{t_g} + \frac{1}{t_{rise}}\right)$$

$$F = \frac{1}{f_c^{3/2}} \approx 1 - \left(\frac{t_g}{\tau_g}\right)^2$$

$$S = \frac{9\pi d}{V} \sqrt{\frac{2\eta^3 x^3}{g\rho}}$$

$$T = \frac{1}{\sqrt{t_g}} \left(\frac{1}{t_g} + \frac{1}{t_{rise}}\right)$$

Route of Charge Calculation. Origin Projects. Data Collecting.

Project: *Millikan_raw data.opj*

Locations: \\engr-file-03\PHYINST\APL Courses\PHYCS401\Students\1. Millikan Oil Drop experiment\Millikan_raw data.opj

Long Name	A(L)	AE(Y)	B(Y)	C(Y)	D(Y)
	parameter label	Units	Par	tg	tr
Units Comments			<i>Parameters, use proper units shown column "Units"</i>	<i>Raw data, falling time</i>	<i>Raw data, rising time</i>
1	η	kg/ms	1.8478E-5		
2	$\Delta\eta/\Delta T$	kg/msCo	4.8E-8		
3	$\rho 1$	kg/m ³	886		
4	$\rho 2$	kg/m ³	1.29		
5	$\rho 1 - \rho 2$	kg/m ³	884.71		
6	g	m/s ²	9.801		
7	p	mmHg			
8	x	m			
9	d	m			
10	V	V			
11	Ta	Co			
12					



Route of Charge Calculation. Origin Projects. Data analysis.

Locations: \\engr-file-03\PHYINST\APL Courses\PHYCS401\Students\1. Millikan Oil Drop experiment

Name	Date modified	Type	Size
Data Analysis for Millikan Oil Drop Experi...	2/22/2008 9:36 AM	Adobe Acrobat D...	59 KB
Millikan_raw data.opj	10/5/2017 4:50 PM	OPJ File	15 KB
Millikan1_calc.opj	2/17/2014 2:52 PM	OPJ File	83 KB
Millikan1_no_calc.opj	9/25/2017 2:03 PM	OPJ File	66 KB
T measurement.opj	10/4/2017 5:09 PM	OPJ File	317 KB

Please make a copy (not move!) of *Millikan1.opj* and *Millikan_raw data.opj* in your personal folder and start to work with your personal copy of the project



Route of Charge Calculation. Origin Project. Data Analysis.

Project *Millikan1.opj*

Long Name	Parameter names	parameter label	Par	rc	tau_g	tg	tr
Units				m		s	s
Comments							
				$r_c[m] = \frac{6.18 \times 10^{-5}}{p[\text{mmHg}]}$	$\tau_g = \frac{2\eta x}{\rho g r_c^2}$	your data	your data
3	Density of oil (kg/m ³)	ρ1	886	8.07843E-8	948.25725	23.07825	31.8955
4	Density of air (kg/m ³)	ρ2	1.29	8.07843E-8	948.25725	20.14243	11.70129
5	Density difference (kg/m ³)	ρ1-ρ2	884.71	8.07843E-8	948.25725	26.97377	22.47531
6	acceleration due to gravity (m/s ²)	g	9.801	8.07843E-8	948.25725	16.34362	16.44208
7	ambient pressure (mmHg)	p	765	8.07843E-8	948.25725	25.93429	25.02886
8	fall/rise distance (m)	x	0.00145			15.34338	9.27446
9	plate separation (m)	d	0.00317			29.3815	19.6161
10	Voltage across the plates (V)	V	500			26.0786	21.2434
11	Air temperature (oC)	Ta	20				

Constants

Plugin your data here

Set Values - [Book1]Sheet1!Col(rc)

Formula wcol(1) Col(A) F(x) Variables

Row (i): From 1 To 1

Col(C) =

$6.18e-5/Col("Par")[7]$

calculations

Recalculate Auto Apply Cancel OK

Col(C) =

$6.18e-5/Col("Par")[7]$

In red - variable parameters
 in blue - data
 in olive - constants

Parameters of the experiment. Depend on exact setup and environment conditions

$$r_c[m] = \frac{6.18 \times 10^{-5}}{p[\text{mmHg}]}$$

P(mmHg) → Col("Par")[7]

index



Route of Charge Calculation. Origin Project. Data Analysis.

Project *Millikan1.opj*

	D(L)	B(X)	F(Y)	G(Y)	C(Y)	E(Y)	H(Y)
Long Name	parameter label	Par	tg	tr	rc	tau_g	F
Units			s	s	m		
Comments			your data	your data	$r_c[m] = \frac{6.18 \times 10^{-6}}{\rho[mmHg]}$	$\tau_g = \frac{2\eta x}{\rho g r_c^2}$	$F = \frac{1}{f_c^{3/2}} \approx 1 -$
1	η	1.8478E-5					
2	$\frac{d\eta}{dT}$	4.8E-8	15.5				
3	ρ_1	886	23.0				
4	ρ_2	1.29	20.1				
5	$\rho_1 - \rho_2$	884.71	26.9				
6	g	9.801	16.3				
7	p	765	25.9				
8	x	0.00145	15.3				
9	d	0.00317	29				
10	V	500	26				
11	Ta	20					
12	$\eta(T) = \eta(25C^\circ) - \frac{d\eta}{dT}(T^\circ - 25)$	1.8478E-5					

Set Values - [Book1]Sheet1!Col("rc")

Formula wcol(1) Col(A) F(x) Variables Options

Row (i): From <auto> To <auto>

Col(C) =

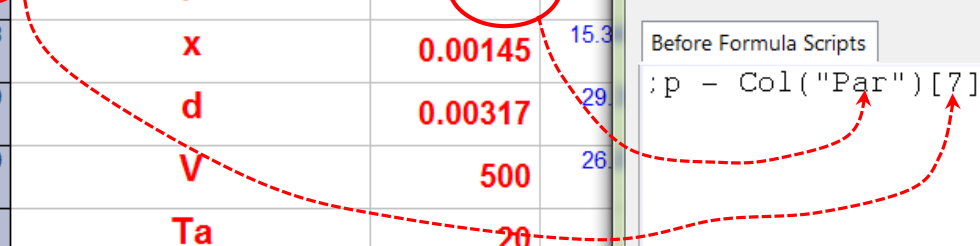
$6.18e-6 / \text{Col}(\text{"Par"})[7]$

Recalculate Auto

Before Formula Scripts

$;\text{p} - \text{Col}(\text{"Par"})[7]$

OK Cancel Apply



Route of Charge Calculation. Origin Project. Data Analysis.

H(Y)	I(Y)	J(Y)	K(Y)	L(Y)
F	S	T	Q=FST	n=Q/1.602e-19
$F = \frac{1}{f_c^{3/2}} \approx 1 - \left(\frac{t_g}{\tau_g}\right)^2 \quad S = \frac{9\pi d}{V} \sqrt{\frac{2\eta^3 x^3}{g\rho}} \quad T = \frac{1}{\sqrt{t_g}} \left(\frac{1}{t_g} + \frac{1}{t_{rise}}\right) \quad Q = F \cdot S \cdot T$				
-	-	-	number of elementary charges	
$Q = F \cdot S \cdot T = \left(\frac{1}{f_c^{3/2}}\right) \frac{9\pi d}{V} \sqrt{\frac{2\eta^3 x^3}{g\rho}} \frac{1}{\sqrt{t_g}} \left(\frac{1}{t_g} + \frac{1}{t_{rise}}\right)$				
-	-	-		
-	-	-		

Follow correct order of calculations: $r_c \rightarrow \tau_g \rightarrow (F, S, T) \rightarrow Q \rightarrow n$

Route of Charge Calculation. Origin Project. Data Analysis.

Project *Millikan1.opj*

Formula wcol(1) Col(A) F(x) Variables

Row (i): From <auto> To <auto>

Col(E) =

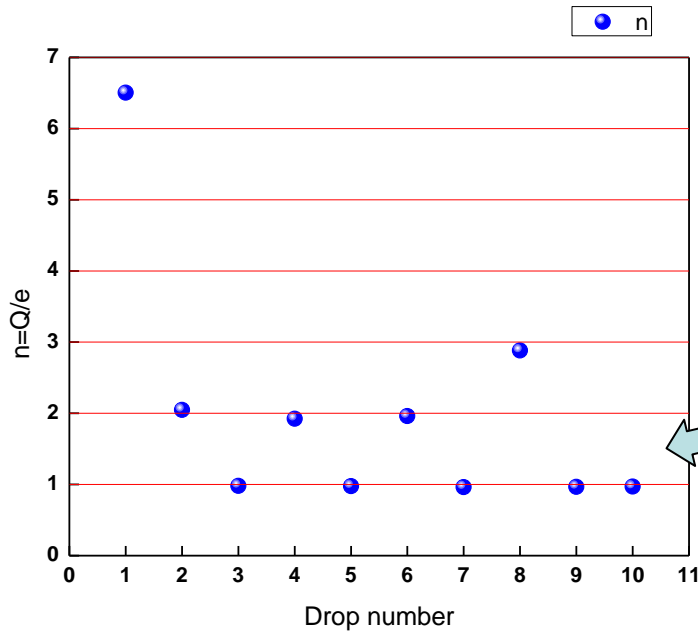
Recalculate None

Before Formula Scripts

```
;(Eta) actual air viscosity - Col("Par")[12] have to be calculated first
;x fall/rise distance      - Col("Par")[8]
;(Rho) density difference  - Col("Par")[5]
;g gravitational constant  - Col("Par")[6]
;rc                        - Col("rc")[1] have to be calculated first
```

**Indexes for parameters in Col("Par")
Actual air viscosity should be calculated manually
before any other calculation**

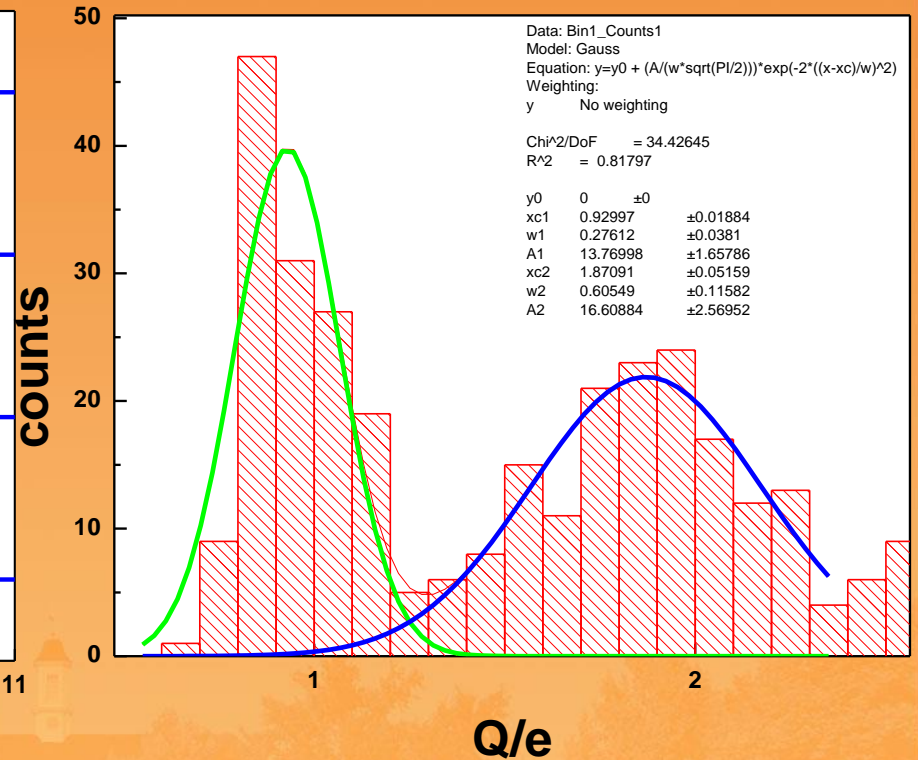
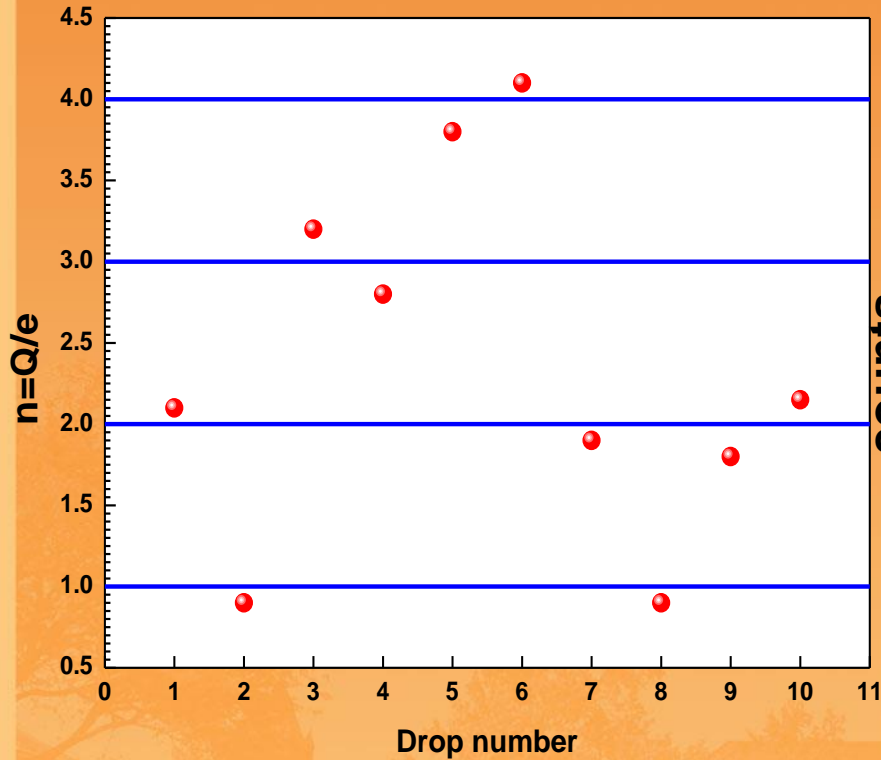
Charge calculation. Origin project.



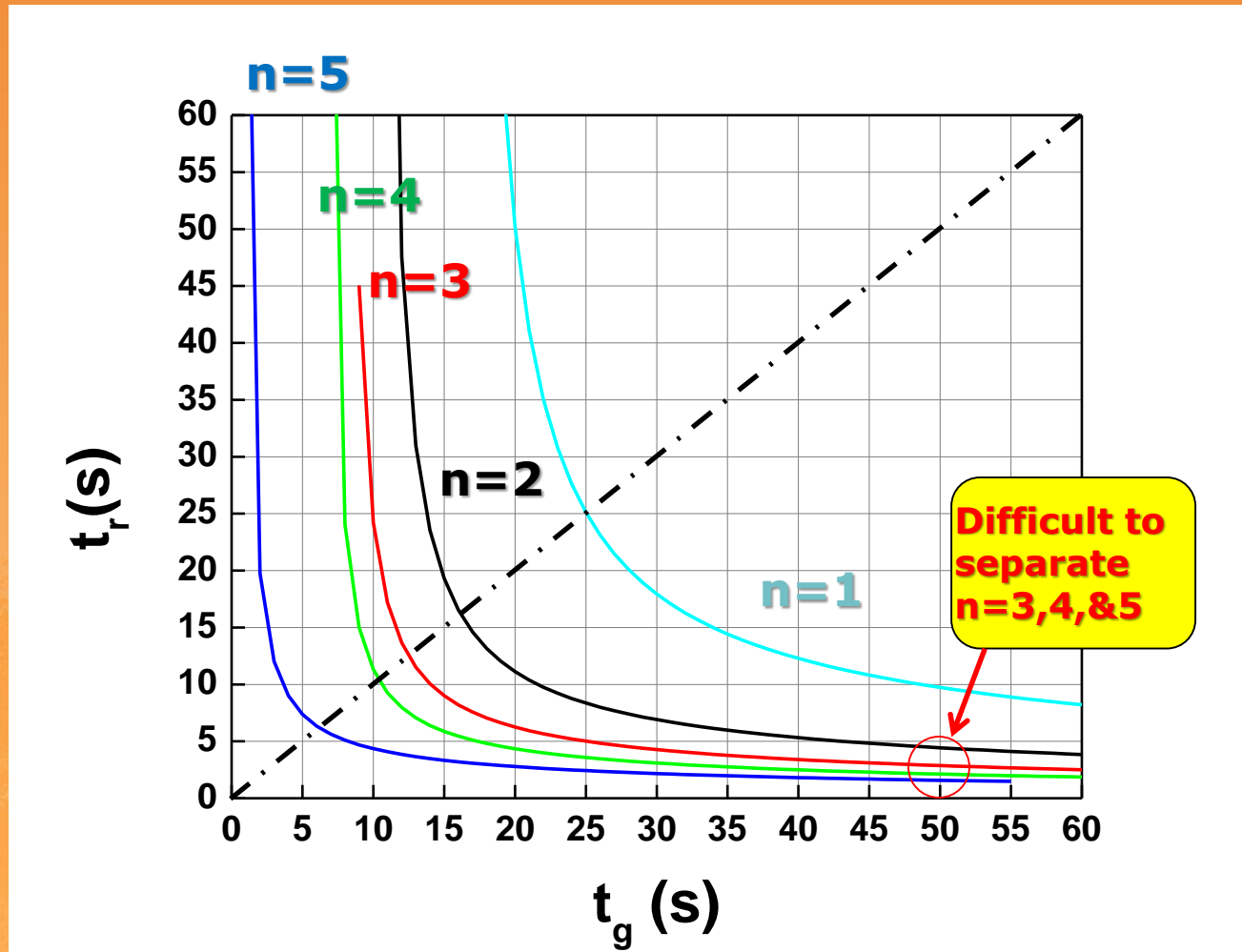
I(Y)	J(Y)	K(Y)	L(Y)
S	T	Q=FST	n=Q/1.602e-19
			c
$S = \frac{9 \pi d}{V} \sqrt{\frac{2 \eta^3 x^3}{g \rho}}$	$T = \frac{1}{\sqrt{t_g}} \left(\frac{1}{t_g} + \frac{1}{t_{rise}} \right)$	Q=F*S*T	number of elementary charges
--	--	1.04196E-19	6.50414
--	--	1.57231E-19	2.0471
--	--	3.08168E-19	0.98147
--	--	1.56458E-19	1.92365
--	--	3.14167E-19	0.97664
--	--	1.54196E-19	1.9611
--	--	4.61924E-19	0.96252
--	--	1.54861E-19	2.88342
--	--	1.55468E-19	0.96668
--	--		0.97046
--	--		
--	--		
--	--		
--	--		
--	--		



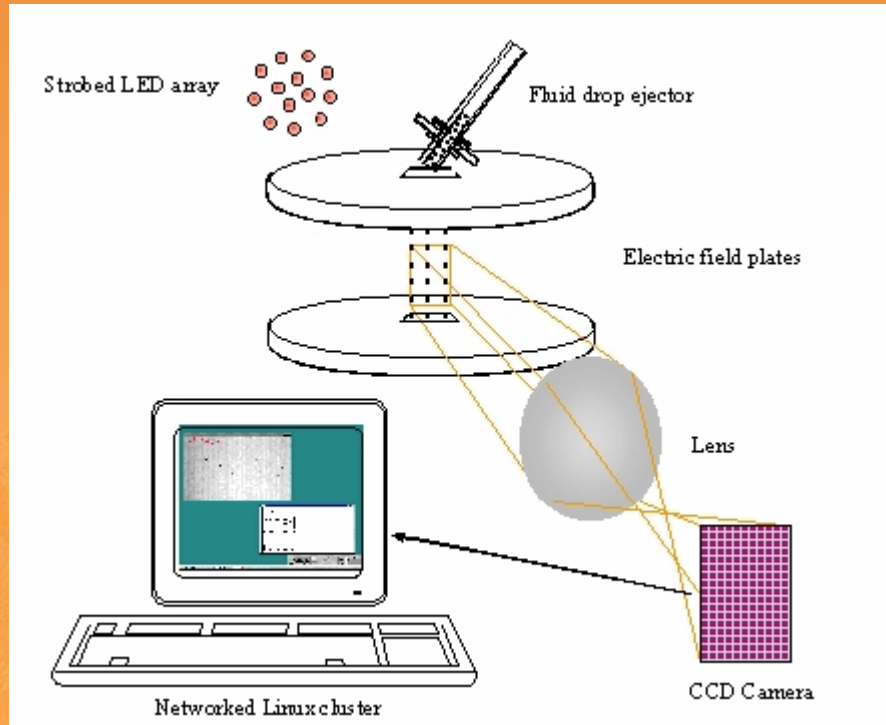
Expected results



Choice of Oil Drops for the Analysis: rise and fall times



Modern experiments at



- **Drop generation rate 1 Hz**
- **Fluid - Dow Corning silicon oil**
- **Number of drops - 17 million**
- **Mass - 70.1 milligrams**
- **Duration - 8 months**

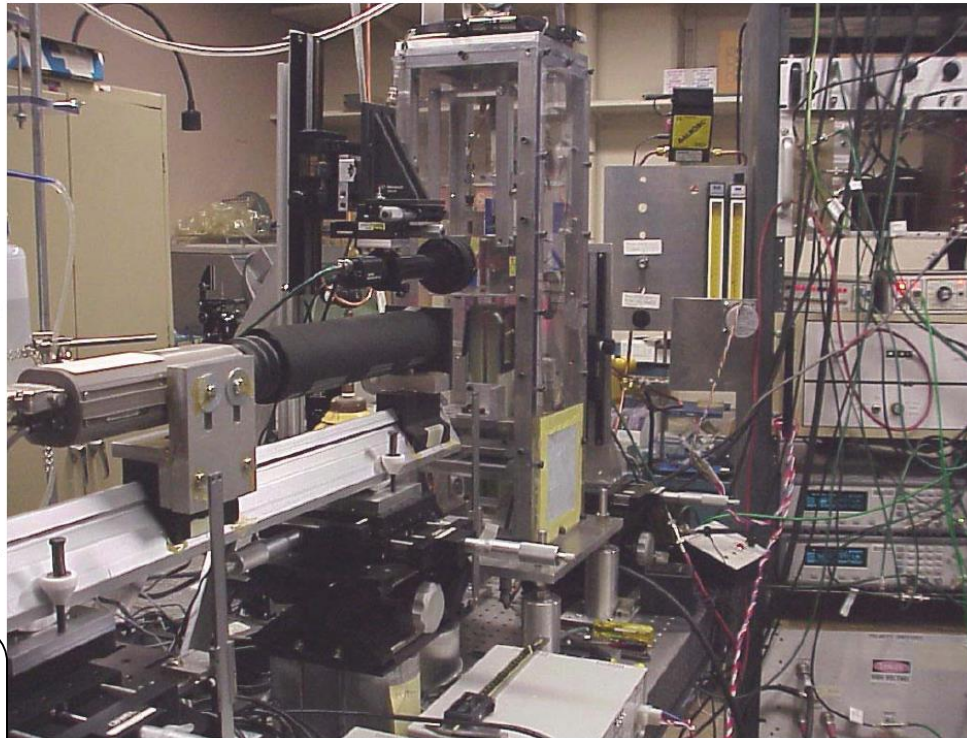


Modern experiments at

Machine vision mediated auto-control of: the average charges of the drops, the fall path of the drops, the upward laminar air flow, and the electronic drive to the drop ejector are new features of this fluid drop charge measurement system.

The experiment is ran from 2004 - 2007.

Charge measurement accuracy achieved is better than $1/24 e$ for drops of up to 26 microns in diameter.



Modern experiments at



Summary as of January 2007.

Total mass throughput for all experiments- 351.4 milligrams of fluid

Total drops measured in all experiments - 105.6 million

No evidence for fractional charge particles was found.

