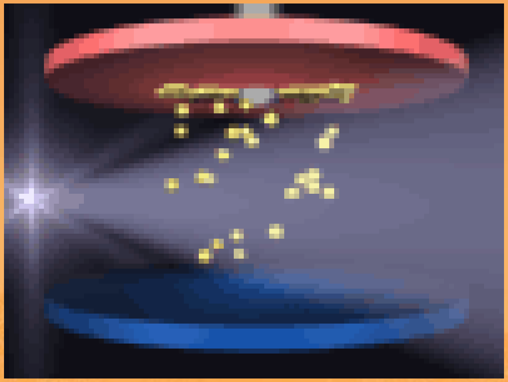
Millikan Oil Drop Experiment

Physics 401, Fall 2019

Eugene V. Colla



Agenda

1. **Measuring of the charge of electron.**
2. **Robert Millikan and his oil drop experiment**
3. **Theory of the experiment**
4. **Laboratory setup**
5. **Data analysis**

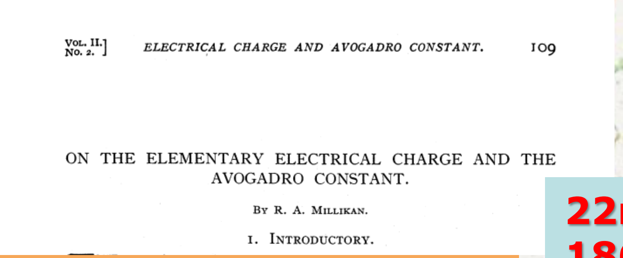
Measuring of the charge of the electron

1. **Oil drop experiment. Robert A. Millikan.. (1909). e=1.5924(17)×10^{-19} C**
2. **Shot noise experiment. First proposed by Walter H. Schottky**
3. **In terms of the Avogadro constant and Faraday constant $e = \frac{F}{N_A}$; F- Faraday constant, N_A - Avagadro constant. Best uncertainty ~1.6 ppm.**
4. **From Josephson ($K_J = \frac{2e}{h}$) and von Klitzing $\left(R_K = \frac{h}{e^2}\right)$ constants**
5. **Recommended by NIST value 1.602 176 565(35) 10^{-19} C**

Robert Millikan. Oil Drop Experiment

ROBERT ANDREWS MILLIKAN
1868-1953

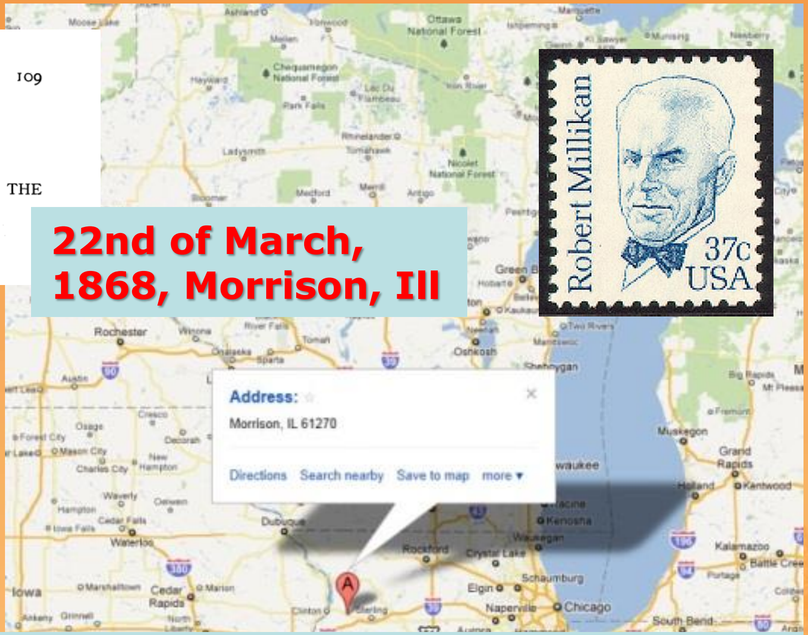
The Nobel Prize in Physics 1923.
Robert A. Millikan "for his work on the elementary charge of electricity and on the photoelectric effect".


Vol. II.] No. 2.] ELECTRICAL CHARGE AND AVOGADRO CONSTANT. 109

ON THE ELEMENTARY ELECTRICAL CHARGE AND THE AVOGADRO CONSTANT.

By R. A. Millikan.

I. Introductory.

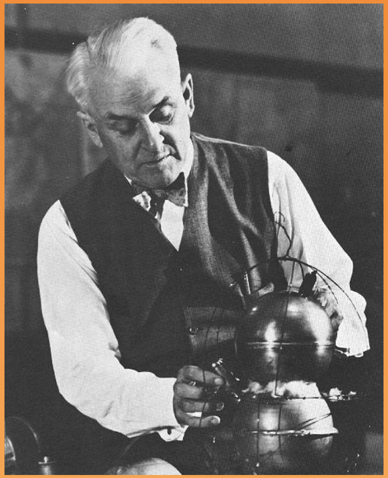


22nd of March, 1868, Morrison, Ill

University of Chicago

On January 26, 1982, he was honored by the United States Postal Service with a 37¢ Great Americans series (1980–2000) postage stamp.

Robert Millikan. Oil drop experiment



ROBERT ANDREWS MILLIKAN
1868-1953

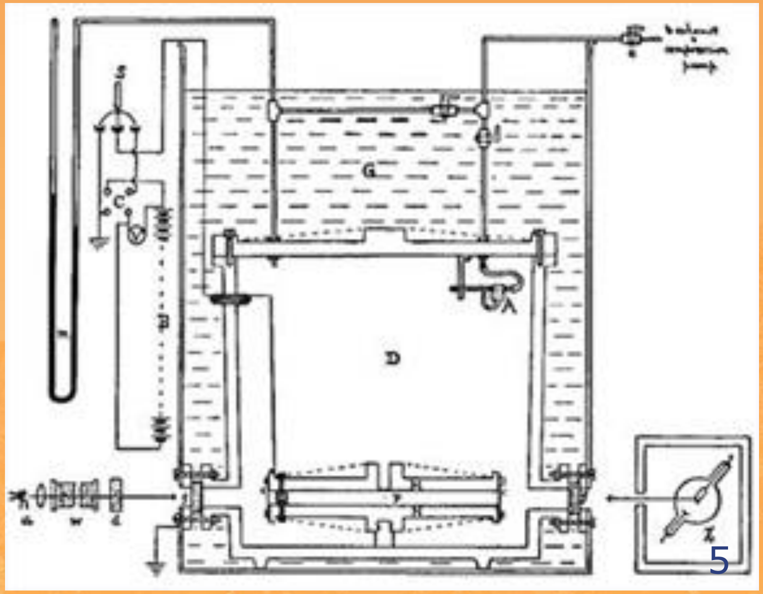


Diagram and picture of apparatus

Oil Drop Experiment.

Motivation:

Measurement of the magnitude of the electron charge!

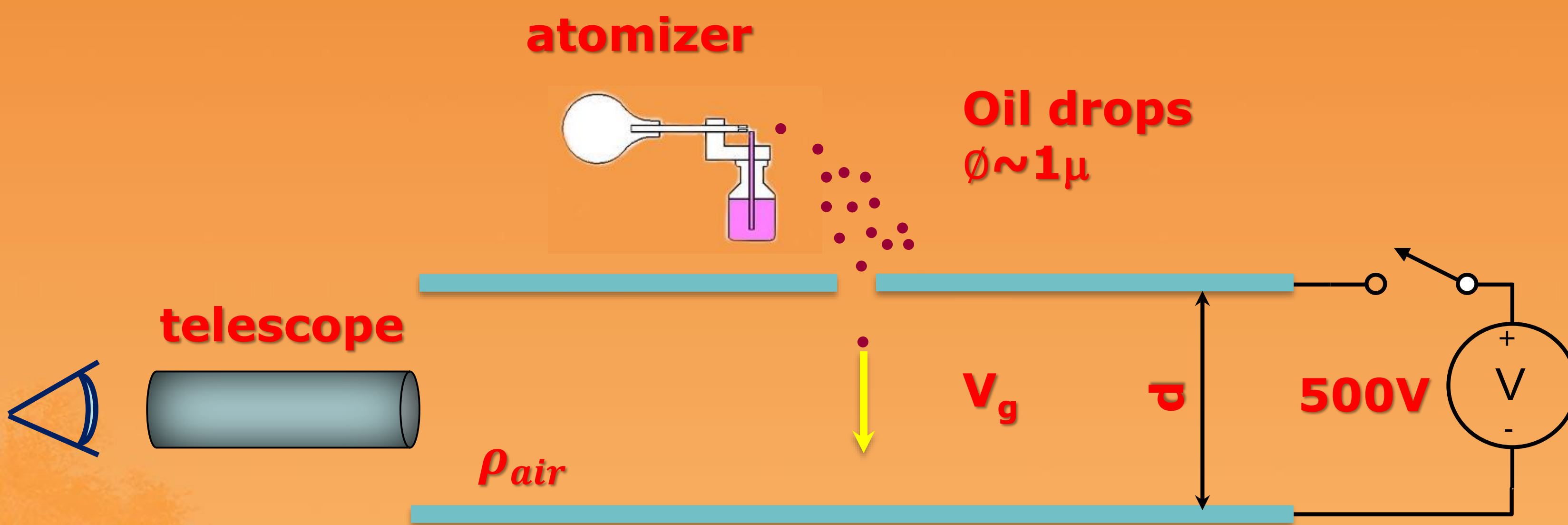
Demonstrate that the electron charge is quantized!

PASCO

Measure the charge of an electron to ±3%

Picture of the PASCO setup

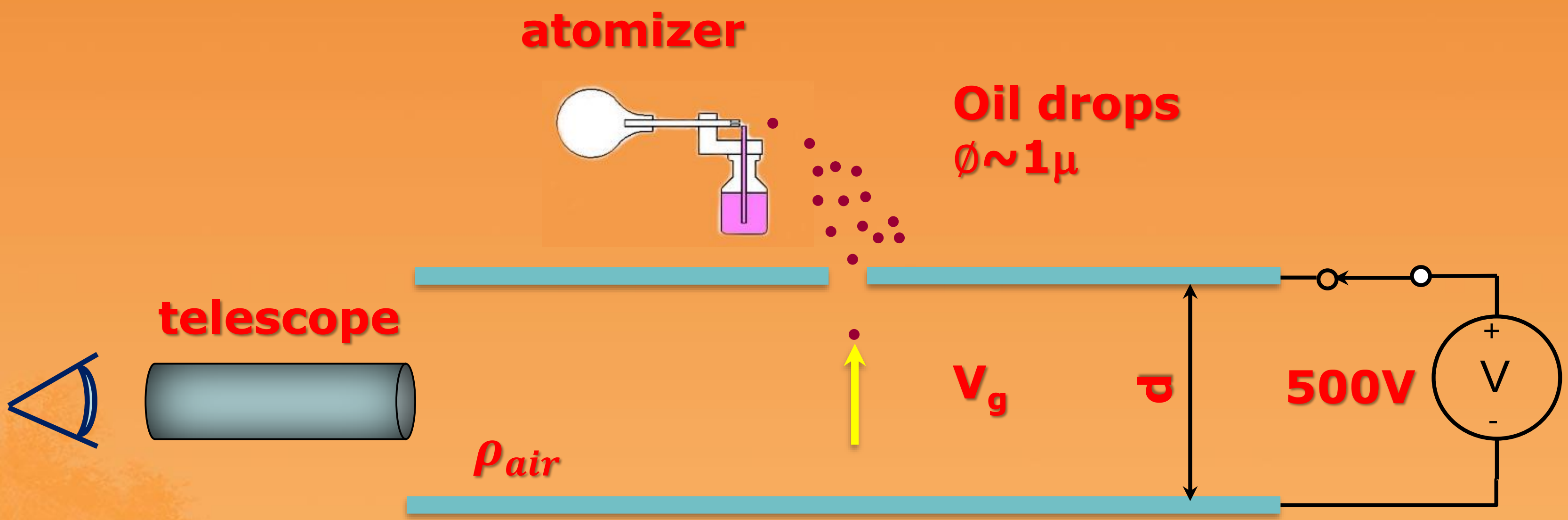
Oil Drop Experiment.



Forces on the oil drop:

1) Gravity + buoyant force (air displaced by oil drop)
2) Drag force of the oil drop in the air

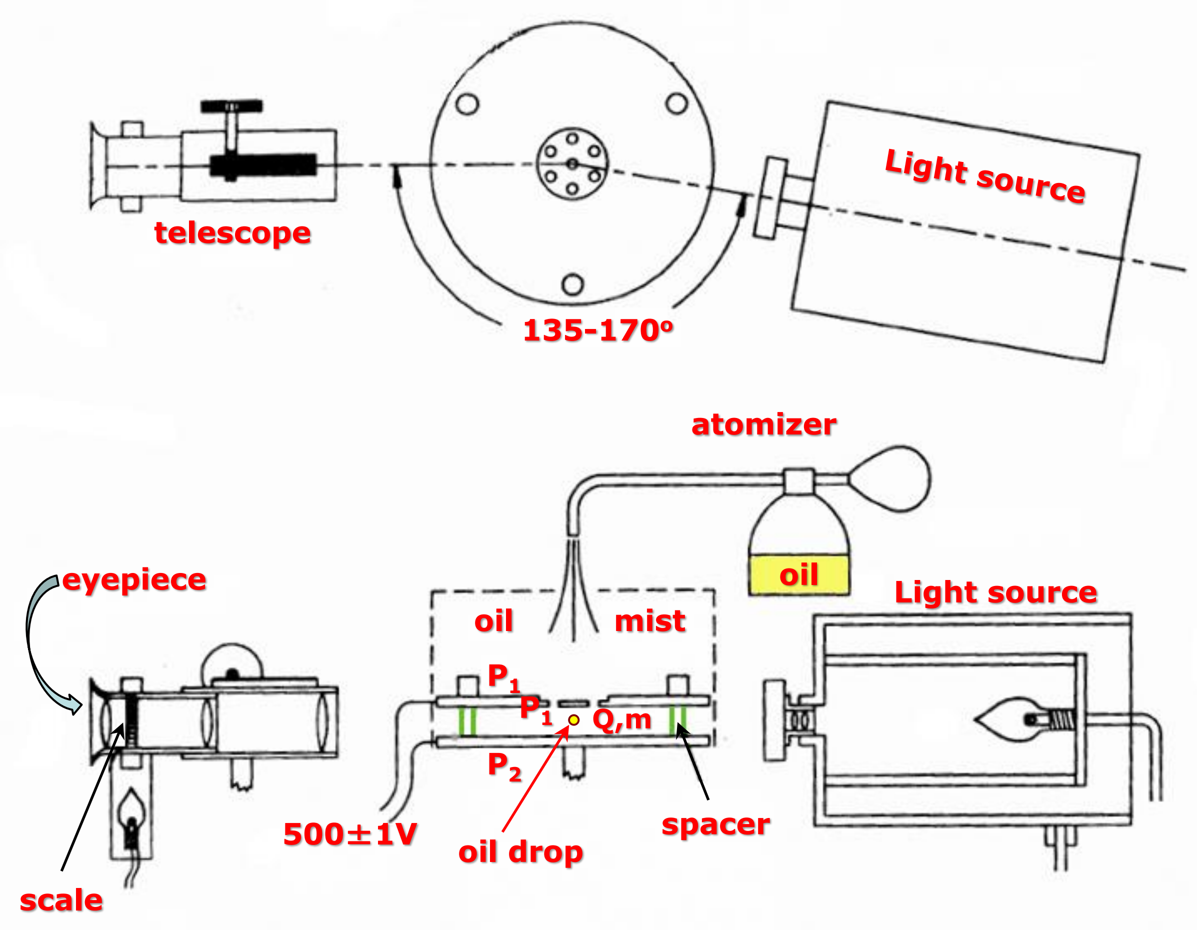
Oil Drop Experiment.



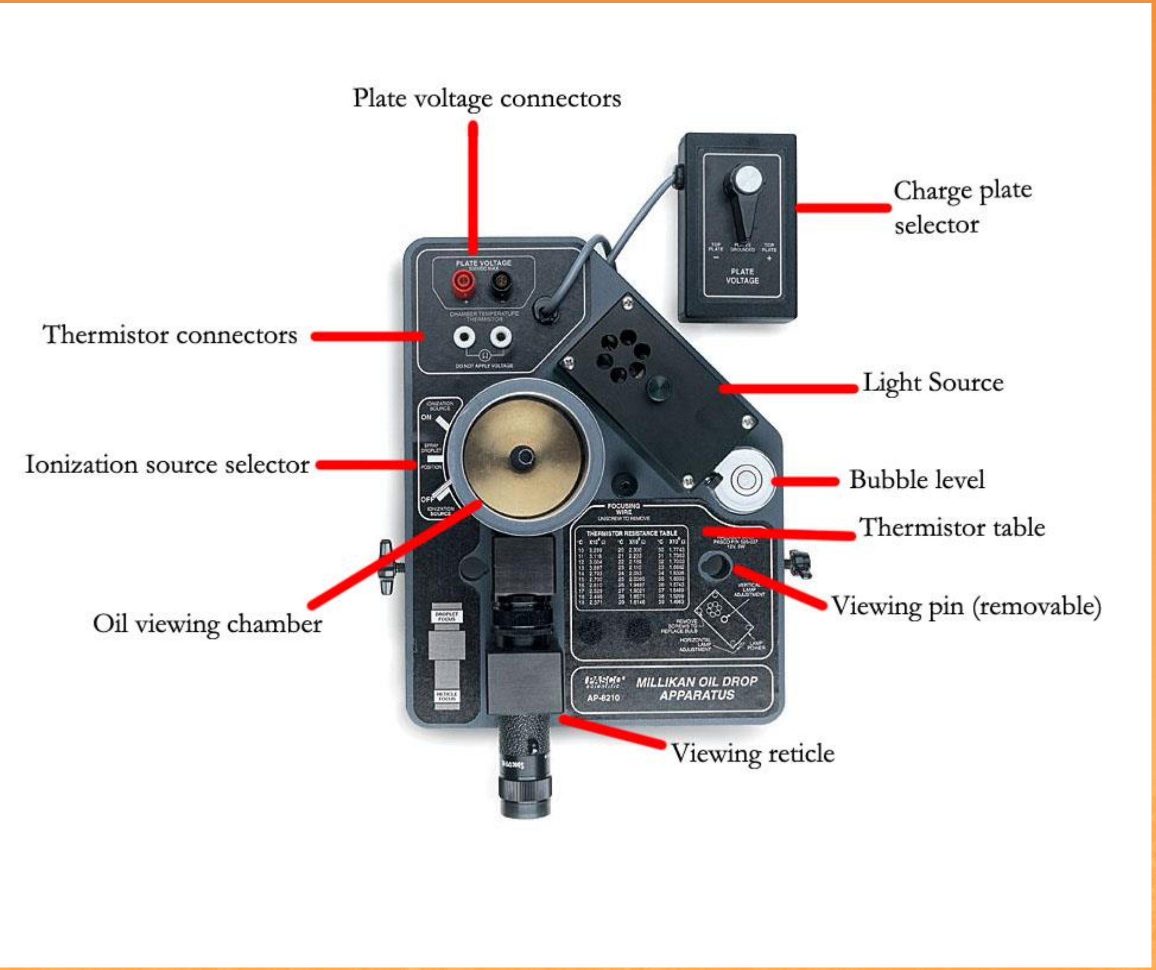
Forces on the oil drop:

1) Gravity + buoyant force (air displaced by oil drop)
2) Drag force of the oil drop in the air
3) Electric force on oil drops which carry charge Q

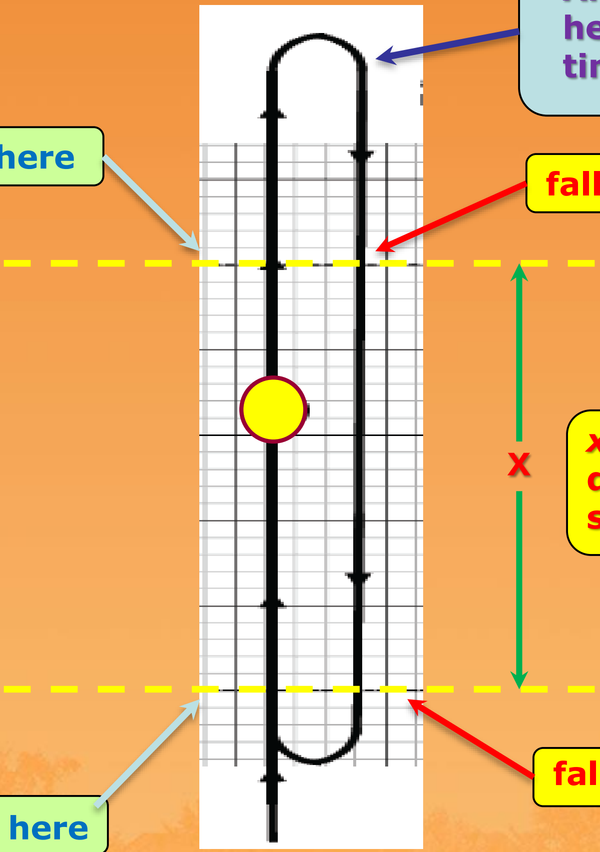
Apparatus. Schematic Layout



Apparatus: Actual Setup



What is Measured



Allow drop to "undershoot" here before starting next rise time experiment

rise time measurement stops here

fall time measurement starts here

fall time t_g

rise time t_{rise}

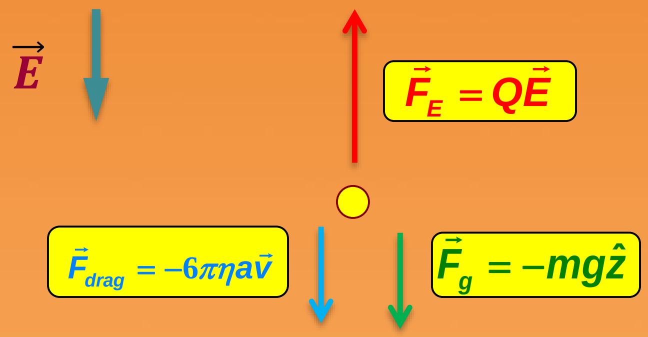
x

x = fall distance = rise distance. x must be the same for all drops!

fall time measurement stops here

rise time measurement starts here

Balance of Forces: Newton's Law



- ***a*** : radius of drop
- **ρ : density $\rho = \rho_{oil} - \rho_{air}$**
- ***v*** **: velocity of oil drop**
- ***Q*** **: charge of oil drop**
- ***E*** **: electric field *E=V/d***
- ***V : Voltage across plates***
- ***η*** **: viscosity of air**
- ***g : gravitational const.***

$$\vec{F}_g = -mg\hat{z} \quad (1)$$

$$\vec{F}_{drag} = -6\pi\eta a\vec{v} \quad (2)$$

$$\vec{F}_E = Q\vec{E} \quad (3)$$

$$\vec{F} = m\frac{d\vec{v}}{dt} = \vec{F}_g + \vec{F}_{drag} + \vec{F}_E$$

Forces on the oil drop:

(1) Gravity + buoyant force (air displaced by oil drop)

(2) Drag force of the oil drop in the air

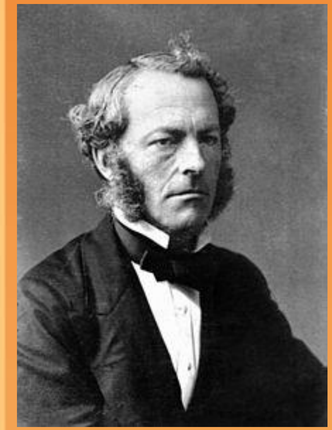
(3) Electric force on oil drops which carry charge Q

Particle reached terminal velocity $\frac{d\vec{v}}{dt} = 0$

$$\vec{F}_g + \vec{F}_{drag} + \vec{F}_E = 0$$

1μ size particle reaches the terminal velocity in ~10^{-5}s

Modification to Stokes Law



George Gabriel Stokes (1819-1903)

$$\vec{F}_{drag} = -6\pi\eta a\vec{v}$$

For small particle radius (a<15μ) Stokes law need to be corrected. This correction was derived by E. Cunningham.

$$\vec{F}_{drag} = -6\pi\eta \frac{a}{f_c}\vec{v}$$

Ebenezer Cunningham (1881-1977)

$$f_c = 1 + A\frac{\lambda}{a} + B\frac{\lambda}{a}e^{-c\frac{a}{\lambda}}, \quad A = 1.246,\ B = 0.42,\ C = 0.78$$

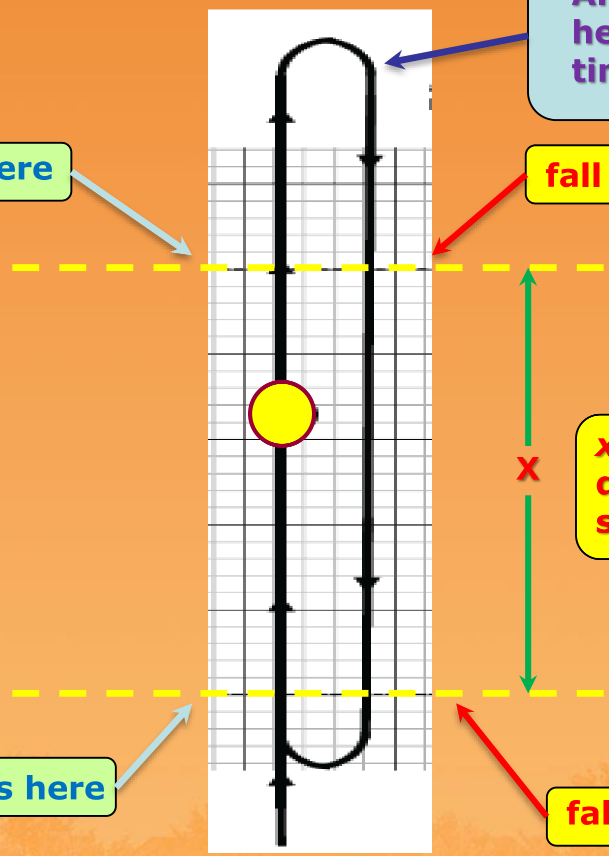
$$f_c \approx 1 + A\frac{\lambda}{a} = 1 + \frac{r_c}{a} \approx 1.1, \quad \text{for } a \approx 10^{-6}\text{m}, \quad r_c[m] = \frac{6.18\times10^{-5}}{p[\text{mmHg}]}$$

negligible term

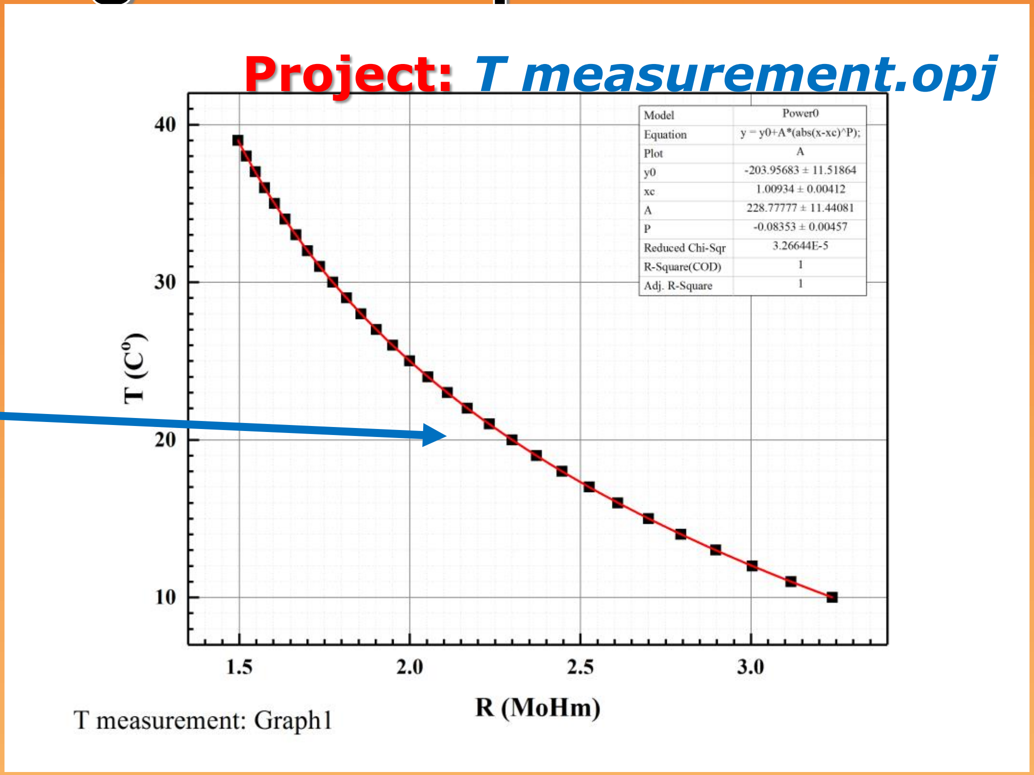
Here a – particle radius; λ – mean free path of the gas molecules

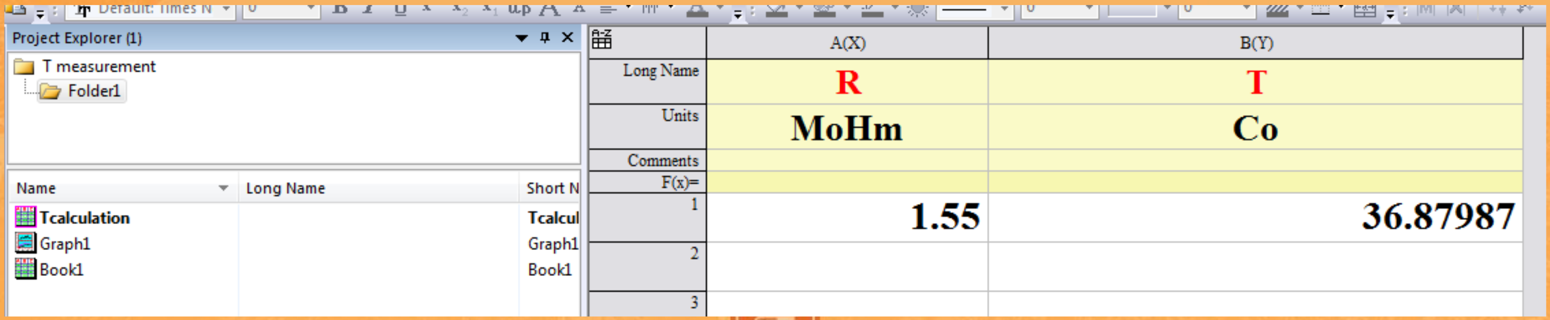
$$\lambda\ [\text{m}] = 6.53\times10^{-8}\frac{760}{p[\text{mmHg}]}$$

We Measure: t_g and t_{rise}



Measuring the Temperature





Solving Newton's Law: $Q(t_g, t_{rise})$

f_c can be found from Newton law equation in the case of V=0 (falling drop)

$$\vec{F}_g + \vec{F}_{drag} = \frac{4}{3} a^3 \rho g - 6\pi\eta \frac{a}{f_c} \vec{v} = 0$$

(see write-up)

$$\frac{1}{f_c^{\frac{2}{3}}} \approx 1 - \left(\frac{t_g}{\tau_g}\right)^{\frac{1}{2}} \;;\; \tau_g = \frac{2\eta x}{\rho g r_c^2}; \quad r_c[m] = \frac{6.18 \times 10^{-5}}{p[mmHg]}$$

Solving Newton's law: $Q(t_g, t_{rise})$

$$Q = n \times e = \frac{1}{f_c^{3/2}} \frac{9\pi d}{V} \sqrt{\frac{2\eta^3 x^3}{g\rho}} \sqrt{\frac{1}{t_g}} \left[\frac{1}{t_g} + \frac{1}{t_{rise}} \right]$$

- *Q* : charge of oil drop
- *n* : number of unpaired electrons in drop
- *e* : elementary charge
- *d* : plate separation
- *V* : Voltage across plates
- ρ : density $\rho = \rho_{oil} - \rho_{air}$
- *η* : viscosity of air
- *g* : gravitational constant
- *x* : drift distance for oil drop
- t_g : fall time
- t_{rise} : rise time

Route of Charge Calculation $Q(t_g, t_{rise})$.

$$\frac{1}{f_c^{\frac{3}{2}}} \approx 1 - \left(\frac{t_g}{\tau_g}\right)^{\frac{1}{2}} \; ; \; \tau_g = \frac{2\eta x}{\rho g r_c^2}; \quad r_c[m] = \frac{6.18 \times 10^{-5}}{p[mmHg]}$$

$$Q = F \bullet S \bullet T = \left(\frac{1}{f_c^{3/2}}\right) \frac{9\pi d}{V} \sqrt{\frac{2\eta^3 x^3}{g\rho}} \frac{1}{\sqrt{t_g}} \left(\frac{1}{t_g} + \frac{1}{t_{rise}}\right)$$

$$F = \frac{1}{f_c^{3/2}} \approx 1 - \left(\frac{t_g}{\tau_g}\right)^{\frac{1}{2}}$$

$$S = \frac{9\pi d}{V} \sqrt{\frac{2\eta^3 x^3}{g\rho}}$$

$$T = \frac{1}{\sqrt{t_g}} \left(\frac{1}{t_g} + \frac{1}{t_{rise}}\right)$$

Route of Charge Calculation. Origin Projects. Data Collecting.

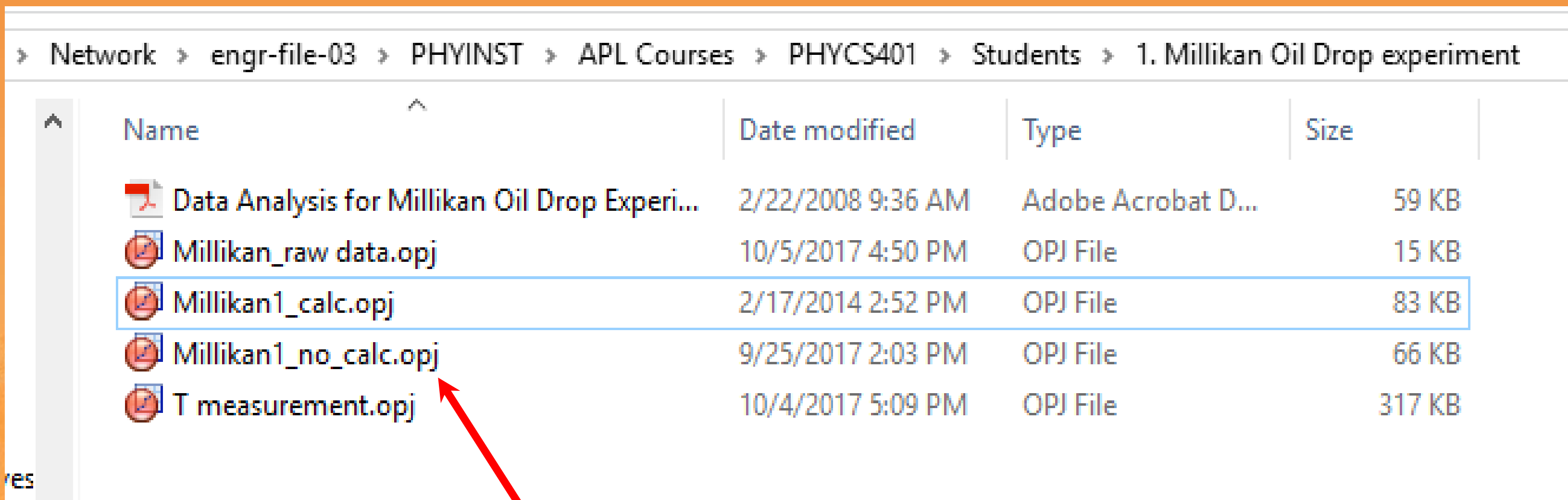
Project: ***Millikan_raw data.opj***

Locations: **\\engr-file-03\PHYINST\APL Courses\PHYCS401\Students\1. Millikan Oil Drop experiment\Millikan_raw data.opj**

	A(L)	AE(Y)	B(Y)	C(Y)	D(Y)
Long Name	parameter label	Units	Par	tg	tr
Units				s	s
Comments			*Parameters, use proper units shown column "Units"*	*Raw data, faling time*	*Raw data, rising time*
1	η	kg/ms	1.8478E-5		
2	Δη/ΔT	kg/msCo	4.8E-8		
3	ρ1	kg/m^3	886		
4	ρ2	kg/m^3	1.29		
5	ρ1–ρ2	kg/m^3	884.71		
6	g	m/s^2	9.801		
7	p	mmHg			
8	x	m			
9	d	m			
10	V	V			
11	Ta	Co			
12					

Route of Charge Calculation. Origin Projects. Data analysis.

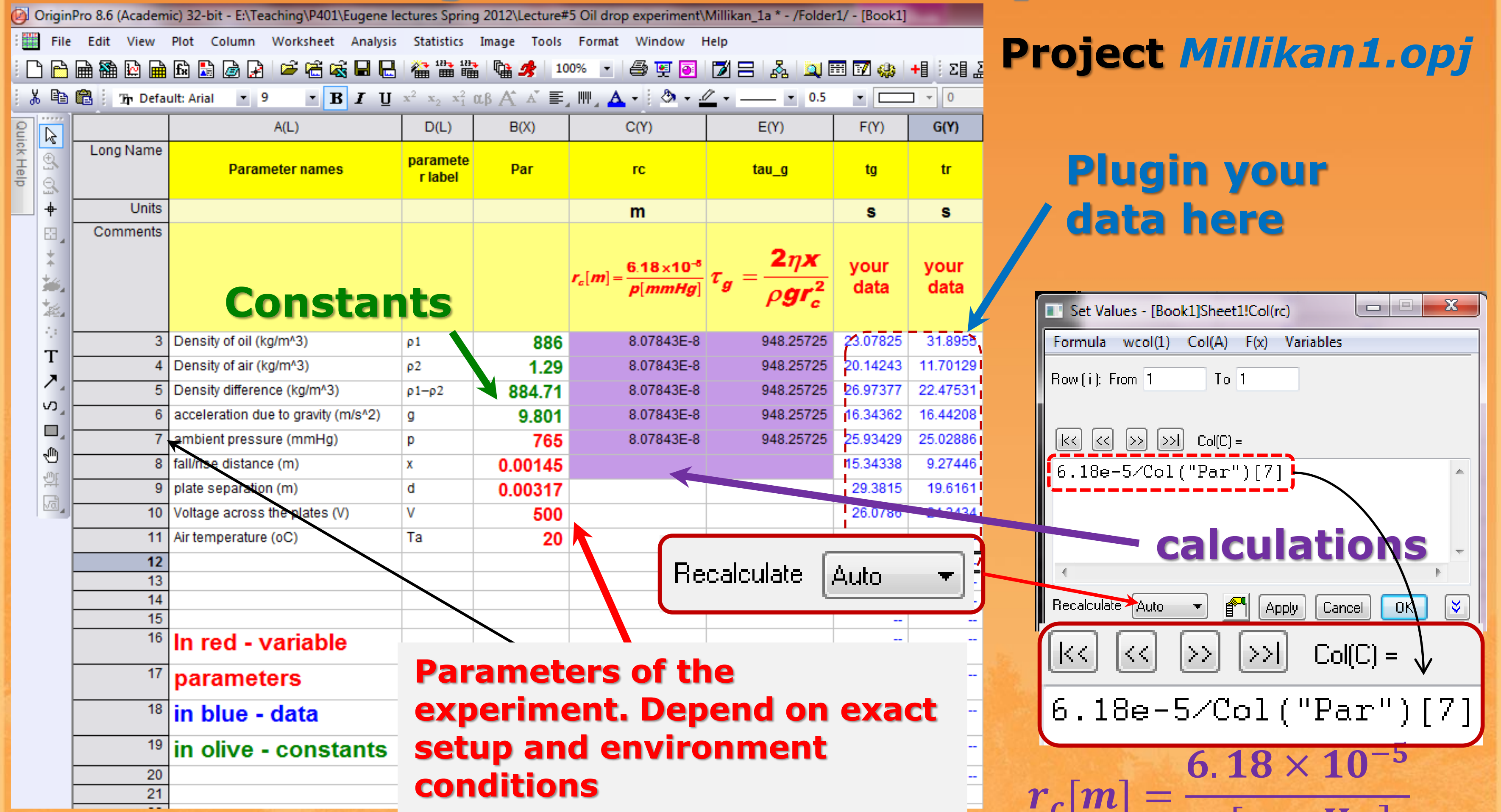
Locations: **\\engr-file-03\PHYINST\APL Courses\PHYCS401\Students\1. Millikan Oil Drop experiment**



Please make a copy (not move!) of *Millikan1.opj* and *Millikan_raw data.opj* in your personal folder and start to work with your personal copy of the project

Route of Charge Calculation. Origin Project. Data Analysis.

Project *Millikan1.opj*



Plugin your data here

calculations

Parameters of the experiment. Depend on exact setup and environment conditions

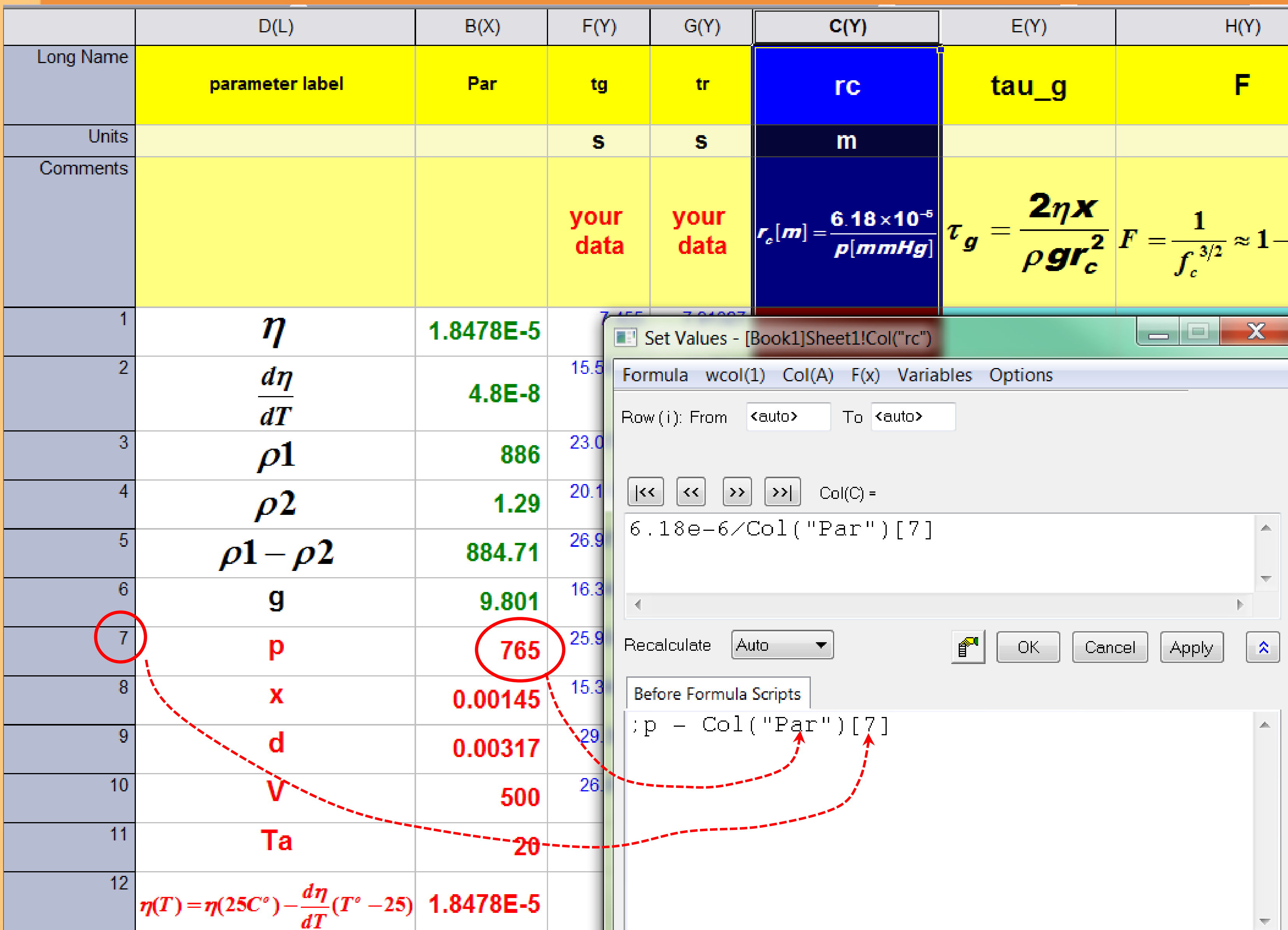
$$r_c[m] = \frac{6.18\times10^{-5}}{p[mmHg]}$$

P(mmHg)→Col("Par")[7]

index

Route of Charge Calculation. Origin Project. Data Analysis.

Project ***Millikan1.opj***



Route of Charge Calculation. Origin Project. Data Analysis.

H(Y)	I(Y)	J(Y)	K(Y)	L(Y)
F	S	T	Q=FST	n=Q/1.602e-19
$F=\frac{1}{f_c^{3/2}}\approx 1-\left(\frac{t_g}{\tau_g}\right)^{\frac{1}{2}}$	$S=\frac{9\pi d}{V}\sqrt{\frac{2\eta^3 x^3}{g\rho}}$	$T=\frac{1}{\sqrt{t_g}}\left(\frac{1}{t_g}+\frac{1}{t_{rise}}\right)$	Q=F*S*T	number of elementary charges
--	--			
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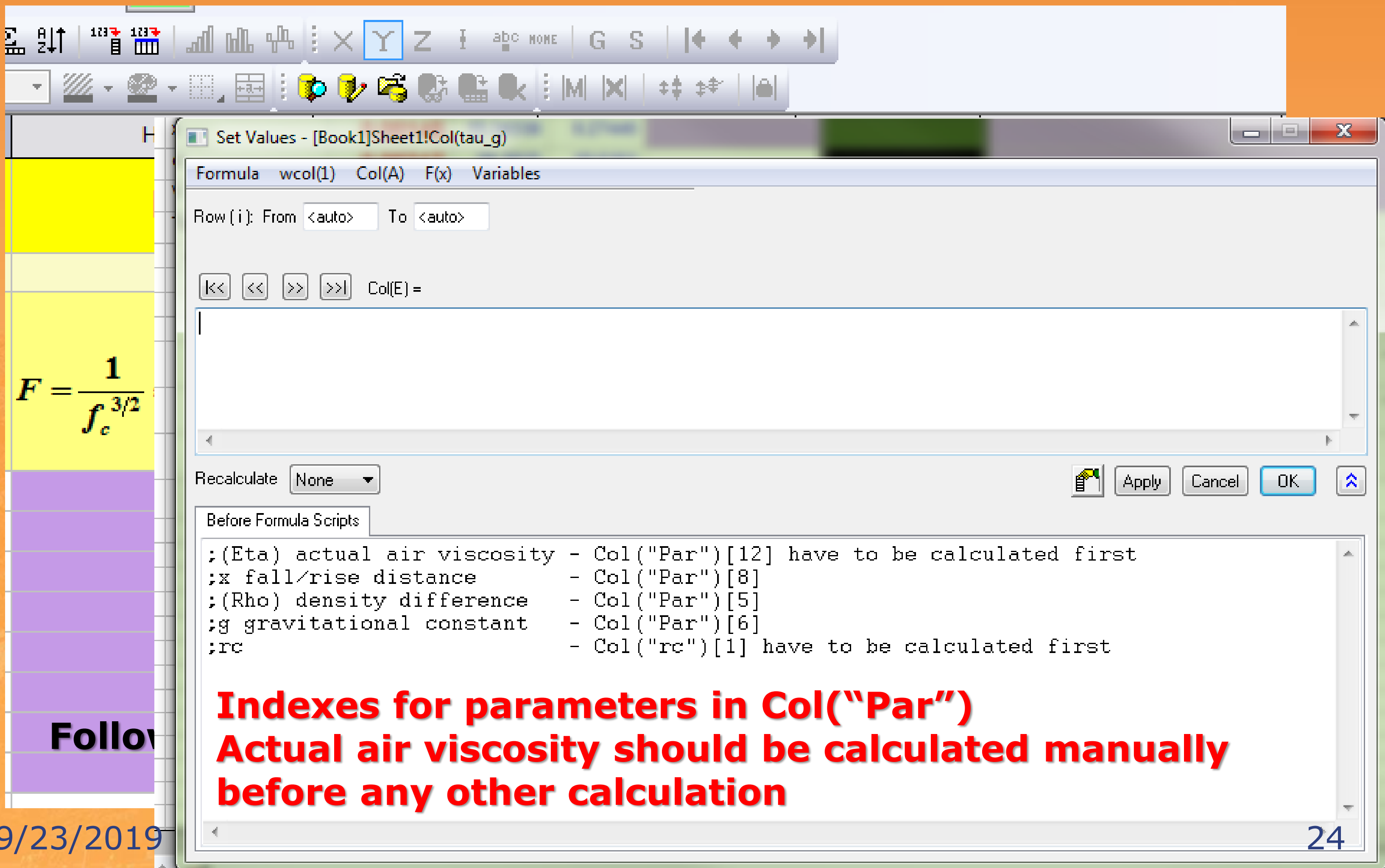
$$Q = F \bullet S \bullet T = \left(\frac{1}{f_c^{3/2}}\right)\frac{9\pi d}{V}\sqrt{\frac{2\eta^3 x^3}{g\rho}}\,\frac{1}{\sqrt{t_g}}\left(\frac{1}{t_g}+\frac{1}{t_{rise}}\right)$$

Follow correct order of calculations: $r_c \rightarrow \tau_g \rightarrow (F,S,T) \rightarrow Q \rightarrow n$

Project *Millikan1.opj*

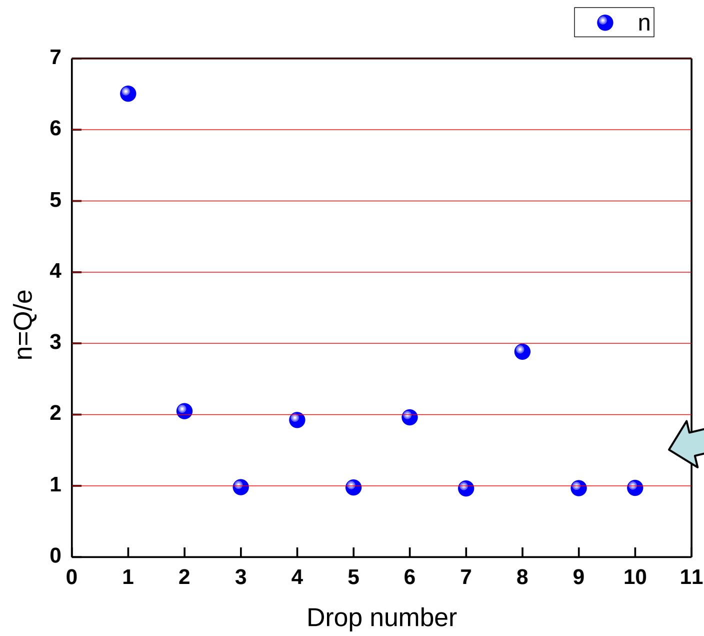
Route of Charge Calculation. Origin Project. Data Analysis.

Project *Millikan1.opj*



Indexes for parameters in Col("Par")
Actual air viscosity should be calculated manually before any other calculation

Charge calculation. Origin project.



	I(Y)	J(Y)	K(Y)	L(Y)
	S	T	Q=FST	n=Q/1.602e-19
				c
	$S=\frac{9\pi d}{V}\sqrt{\frac{2\eta^3 x^3}{g\rho}}$	$T=\frac{1}{\sqrt{t_g}}\left(\frac{1}{t_g}+\frac{1}{t_{rise}}\right)$	Q=F*S*T	number of elementary charges
	--		1.04196E-1[illegible]	6.50414
	--		[illegible]945E-19	2.0471
	--		1.57231E-19	0.98147
			3.08168E-19	1.92365
	--		1.56458E-19	0.97664
	--		3.14167E-19	1.9611
	--		1.54196E-19	0.96252
	--		4.61924E-19	2.88342
	--		1.54861E-19	0.96668
	--		1.55468E-19	0.97046
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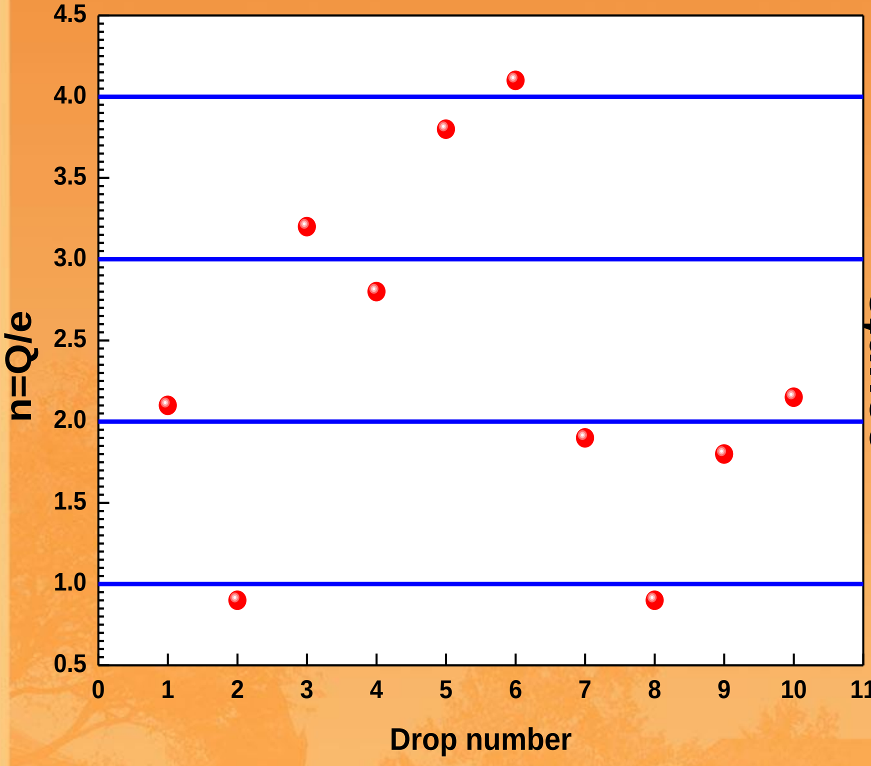
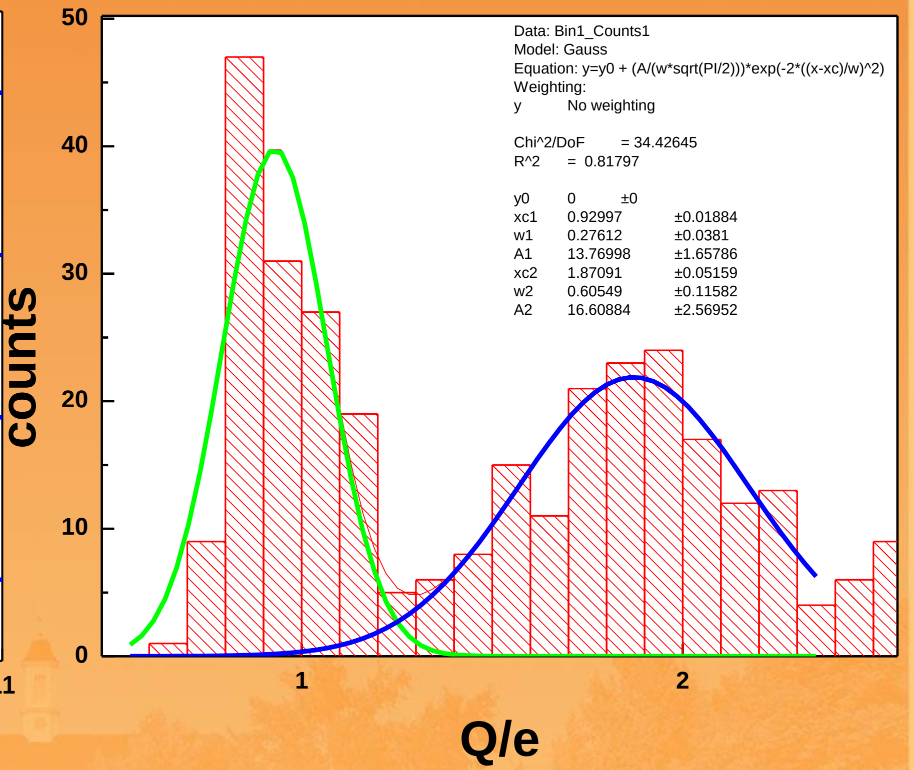
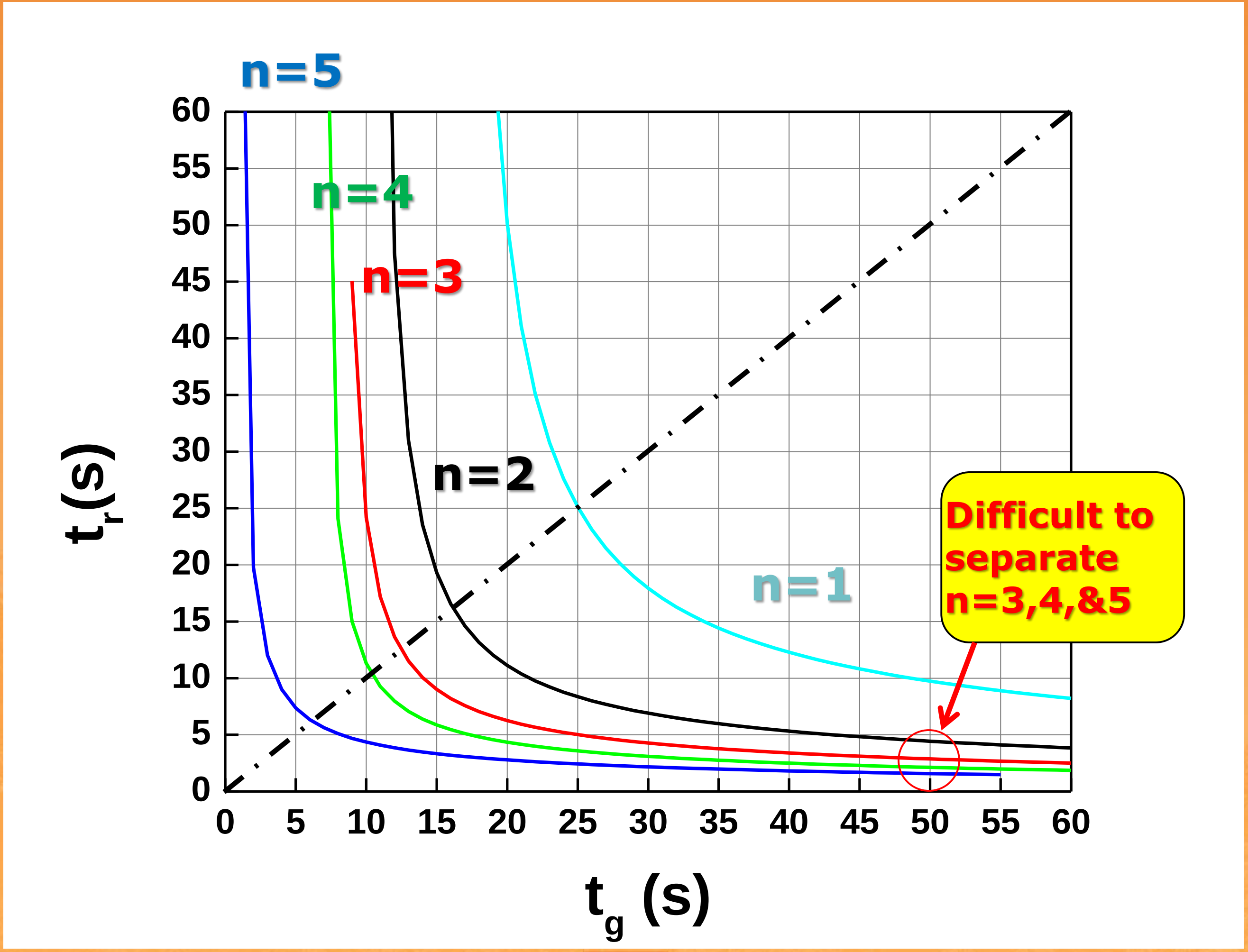
Expected results



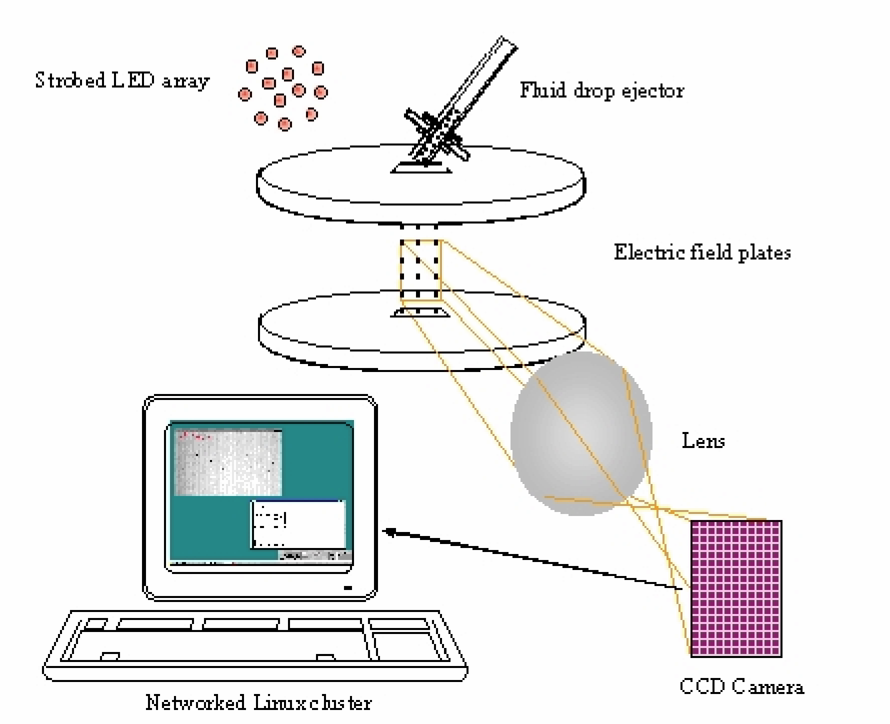


Choice of Oil Drops for the Analysis: rise and fall times



Modern experiments at SLAC

NATIONAL ACCELERATOR LABORATORY



- Drop generation rate 1 Hz
- Fluid - Dow Corning silicon oil
- Number of drops - 17 million
- Mass - 70.1 milligrams
- Duration - 8 months

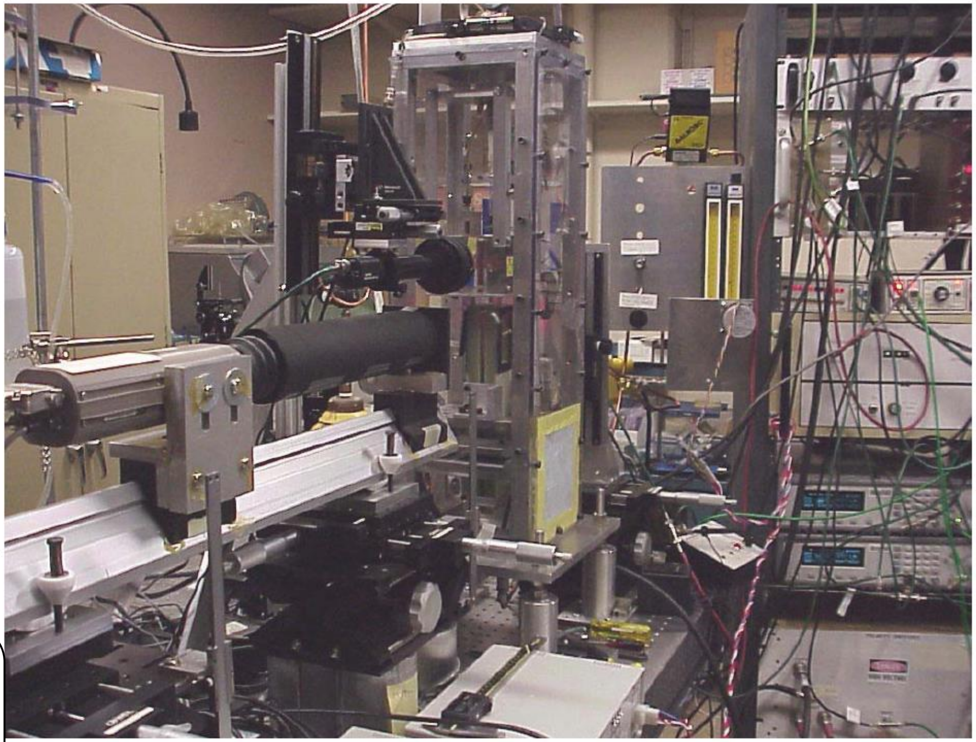
Modern experiments at SLAC

NATIONAL ACCELERATOR LABORATORY

Machine vision mediated auto-control of: the average charges of the drops, the fall path of the drops, the upward laminar air flow, and the electronic drive to the drop ejector are new features of this fluid drop charge measurement system.

The experiment is ran from 2004 - 2007.

Charge measurement accuracy achieved is better than 1/24 e for drops of up to 26 microns in diameter.



Modern experiments at SLAC

Summary as of January 2007.
Total mass throughput for all experiments- 351.4 milligrams of fluid
Total drops measured in all experiments - 105.6 million
No evidence for fractional charge particles was found.